

An evaluation of stress intensity factor solutions for a corner crack at a hole

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ERSI and QinetiQ Australia
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- The author/presenter thanks the participants in the round robins, and the organisers and integrators of the Engineered Residual Stress Implementation (ERSI) group who made significant contributions to this work.
- Thanks to Bob Pilarczyk, Hill Engineering, for his hard work with ERSI and co-ordinating the current effort, and to Jesse Guymon, Hill Engineering, for his work co-ordinating the associated report
- Thanks to the participants who made submissions:

Börje Andersson, BARE Research

Joe Cardinal, Southwest Research Institute

Jim Harter, LexTech Inc.

Adrian Loghin, Simmetrix Inc.

Sebastian Nervi, Engineering Software Research and Development (ESRD) Inc.

Jim Newman, Mississippi State University

Per Nordlund, Hexagon MI

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- Results
- Investigation and follow-up work
- Conclusion

Background

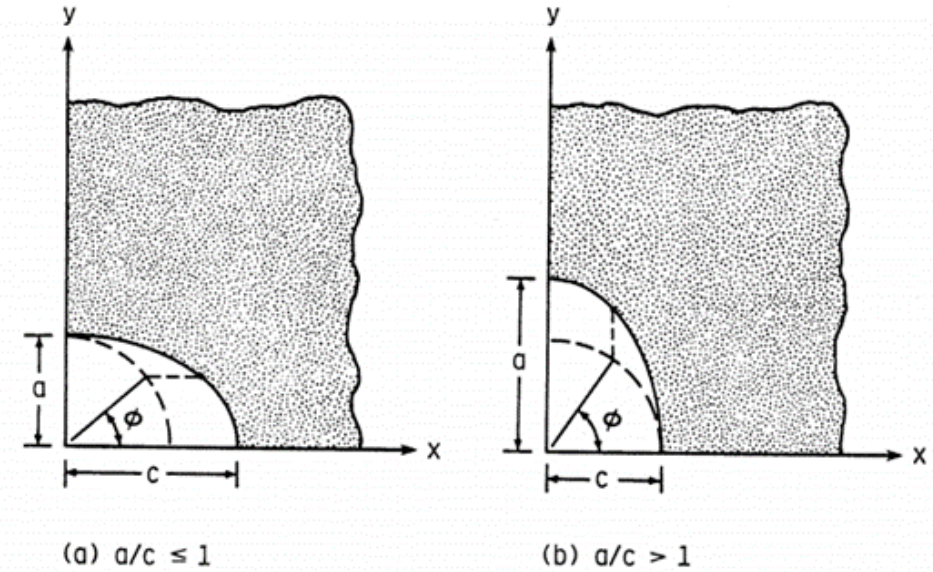
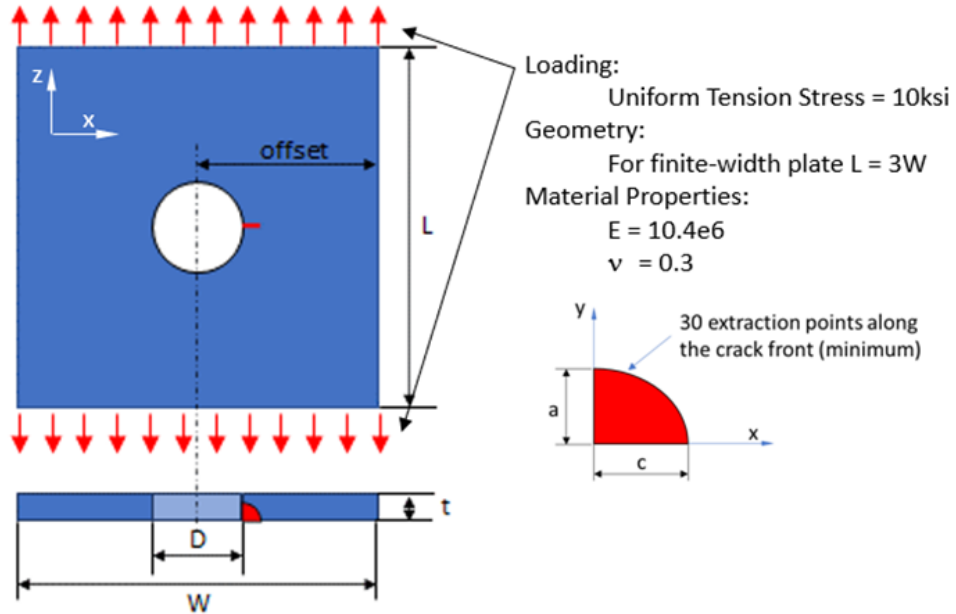
- Engineered Residual Stress Implementation (ERSI) working group established to:
 - Develop a roadmap for the implementation of engineered deep residual stress for fatigue and fracture critical aerospace components
 - Highlight the gaps in the state-of-the-art
 - Define the most effective ways to document requirements and guidelines for a more holistic, physics-informed method for fleet-wide implementation
- Round Robin conducted in 2017 to evaluate methods to predict/analyse crack growth in cold-worked holes suggested that inaccuracy in handbook SIF solutions may be an issue
- Another Round Robin was therefore conducted to evaluate alternative methods and tools for calculating stress intensity factors around the crack front for a common configuration, i.e. a single corner crack at an open hole in a finite-width plate
- Seven cases were devised, and nine submissions were received from eight participants

ERSI ENGINEERED RESIDUAL STRESS IMPLEMENTATION

Aim

- Evaluate the accuracy and efficiency of the K-solutions determined by a broad range of analysts for seven representative scenarios
- Identify areas where improvements can be made and make appropriate recommendations

Round Robin Description



Case	Surface Crack Length (c) (inch)	Bore Crack Length (a) (inch)	Corner Crack Configuration	Width (inch)	Thickness (inch)	Hole Diameter (inch)	Hole Offset (inch)
1	0.050	0.050	Double Symmetric	100.00	0.25	0.50	50.00
2	0.050	0.050	Single	100.00	0.25	0.50	50.00
3	0.050	0.050	Single	4.00	0.25	0.50	2.00
4	0.050	0.050	Single	4.00	0.25	0.50	0.60
5	0.050	0.050	Single	1.20	0.25	0.50	0.60
6	0.050	0.075	Single	100.00	0.25	0.50	50.00
7	0.100	0.050	Single	100.00	0.25	0.50	50.00

Summary of submissions

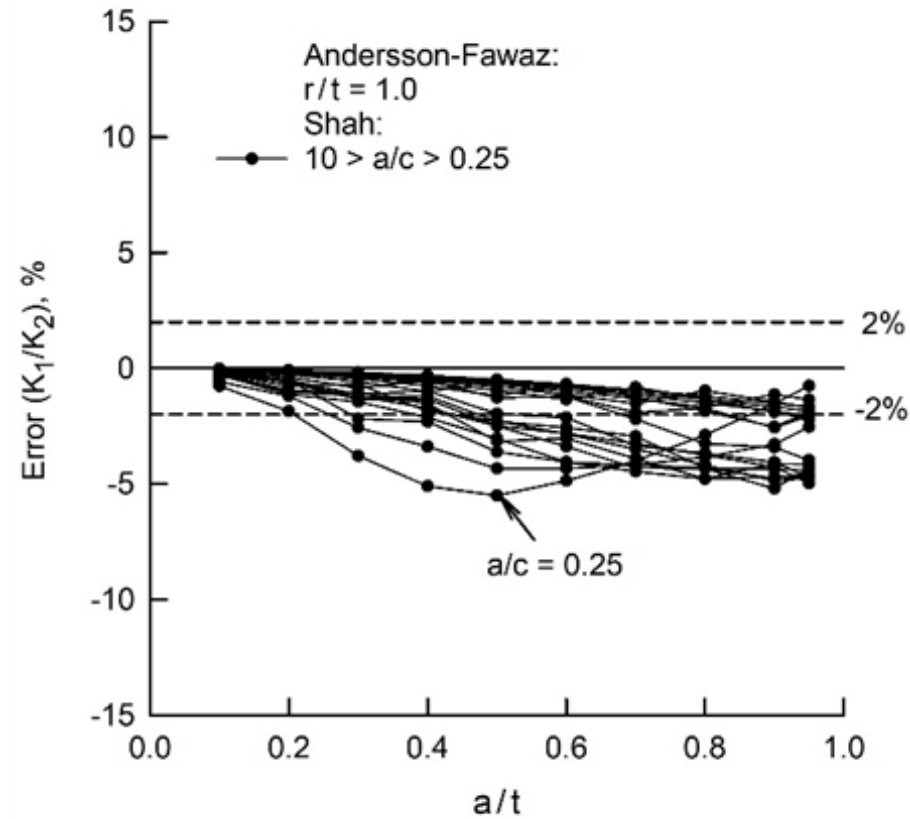
Submission #	Title	SIF solution source	Single Corner Crack Correction (Cases 2, 3, 4, 5, 6, 7)	Finite Width Correction (Cases 3, 4, 5)	Offset Hole Correction (Cases 4, 5)
1	Fawaz-Andersson Solutions, AFGROW	Fawaz-Andersson [4] (as implemented in AFGROW Advanced Model)	n/a	Newman correction [7]	Harter correction [5]
2	Newman-Raju Fit to Fawaz-Andersson	Updated equations by Newman [6] based on fit to Fawaz-Andersson solutions [4]	Shah-Newman Correction (2020)	Newman correction [7]	<ul style="list-style-type: none"> center hole (conservative option) Kt match approach
3	Newman-Raju (1986)	1986 Newman-Raju solution [7]	Shah correction	Newman correction [7]	Kt match approach
4	NASGRO (CC04 & CC02): Newman-Raju	1986 Newman-Raju solution [7] (as implemented in NASGRO CC04)	Shah correction (as implemented in NASGRO CC02)	NASGRO CC02 [9]	NASGRO CC02 [9]
5	NASGRO (CC16): Fawaz-Andersson	Fawaz-Andersson solutions [4] (as implemented in NASGRO CC16)	n/a	Modified version [10] of the Newman correction [7]	Harter correction [5] (as implemented in NASGRO CC16)
6	Andersson: FEA (2021)	Explicitly modeled each condition utilizing the STRIPE FE-software for the hp-version of the finite element method			
7	SimModeler Crack: FEA (2021)	Utilized SimModeler Crack to create 3D FEMs and compute Mode I SIFs via displacement correlation technique			
8	StressCheck: FEA (2021)	Utilized StressCheck to create 3D FEMs and compute Mode I SIFs			
9	Marc: FEA (2021)	Utilized Marc to create 3D FEMs and compute Mode I SIFs			

Details of submissions

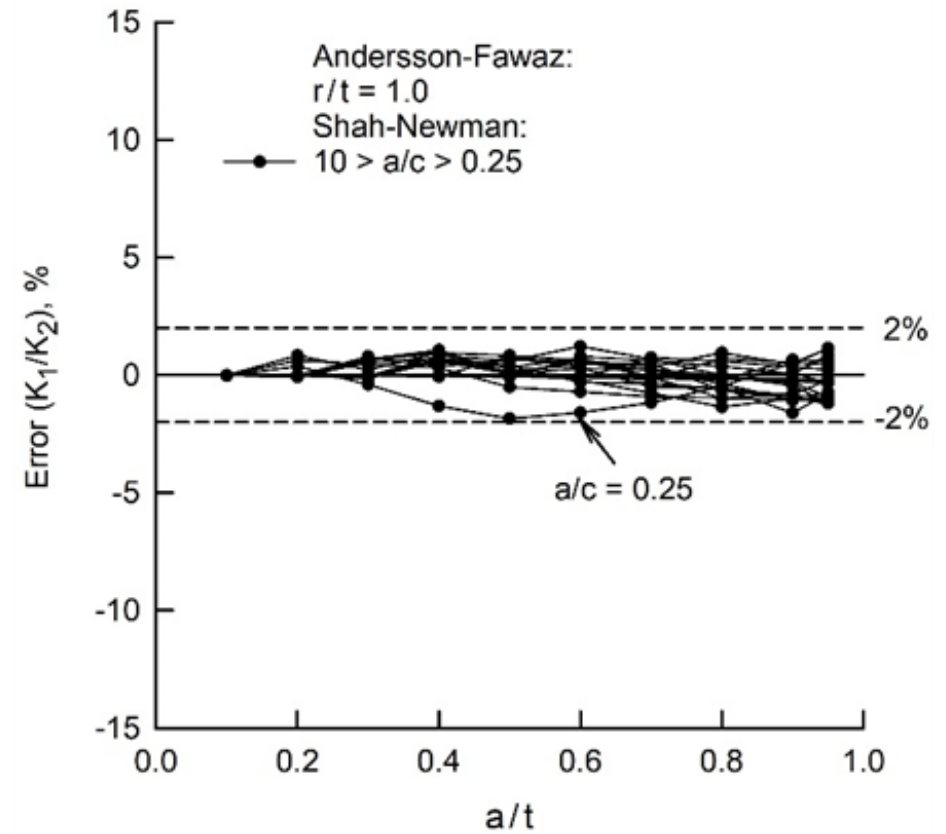
- Fawaz-Andersson 2004 solutions [4], as implemented in AFGROW Advanced Model
- Fawaz-Andersson developed a database of 226,875 solutions for a range of symmetric and unsymmetric corner cracks at a hole for a range of crack aspect ratios ($a/c = 0.1-10$), crack depth to thickness ratios ($a/t = 0.1-0.99$) and for hole radius to sheet thickness ($r/t=1.0$). The r/t for the current work is also 1.0.
- So the range of cases considered in the current work is well within the bounds of the Fawaz-Andersson solutions
- Important to note that the Fawaz-Andersson database of solutions were developed for a very wide plate, effectively an “infinite” width
- Where required (Cases 3, 4 and 5) the finite width correction from Newman-Raju [6] is applied
- Harter offset hole correction applied for case where hole is not central (Case 4)

- Updated curve-fit equations developed by Newman in 2012 [6] based on modifying the Newman-Raju 1986 equations [7] to fit the Fawaz-Andersson 2004 [4] solutions for two symmetric corner cracks at a hole
- Correction for two symmetric cracks to a single crack uses an update to the Shah 1976 [8] correction labelled as the “Shah-Newman” correction. Details in [23]. See plots on next slide.
- Finite-width correction from Newman-Raju 1986 [7] for Cases 3, 4 and 5.
- For Case 4, offset hole, two options were applied. Option 1 conservatively assumed a central hole in a narrow-plate with the width determined by the 0.6 inch offset, for a total width of 1.2 inches. Option 2 was to assume a central hole in an “effective width” plate with the width such that the same stress concentration K_T is produced at the crack site, an effective width of 1.43 inches. Option 2 results presented here.

Sub 2 – Updated double-single crack correction



Original Shah correction (1976)



Updated correction – Shah-Newman 2021

- 1986 Newman-Raju solution [7] for SIF and finite width correction
- Shah 1976 [8] correction from double to single crack for Cases 2-7
- Offset hole Case 4 was accounted for using the Option 2 “ K_T match” approach as described for Submission 2

- NASGRO CC04 solution, which incorporates the 1986 Newman-Raju solution [7]
- For Cases 2-7, the NASGRO CC02 model which corrects for a single corner crack using the Shah 1976 correction [8] was utilized. CC02 correction factors for finite width effects and offset holes used a solution for a through crack from an offset hole in a plate [9].
- These legacy NASGRO solutions (CC02 and CC04) were included in the round robin exercise for comparison purposes but have been superseded by CC16 and are not recommended for use.

- Submission 5 utilized the NASGRO CC16 solution, which incorporates the Fawaz-Andersson 2004 solutions [4].
- Fundamentally based on the original Fawaz-Andersson solutions, CC16 represents the a-tip and c-tip SIFs with single values based on the local maximum (peak value) observed near the surface, which is usually around 2 to 3 degrees, but varies from case to case.
- To correct for a finite width, a modified version of the Newman 1986 finite width correction factor [7] was used [10] where applicable (Cases 3-5).
- To account for offset holes (Case 4), the Harter offset correction from AFGROW [5] is utilized in CC16. For the comparisons in this study, the CC16 results are compared to the local maximum results at the angles reported for the Andersson (2021) solutions.

- **Explicit model using STRIPE FE software using hp-version for each case**
- **STRIPE was developed at the Aeronautical Research Institute of Sweden, FFA, 1984**
- **Using p-values of up to 10, convergence studies showed relative error in K at arbitrary points along the crack front of less than 0.03%, and for Case 2, error of around 0.01%**
- **The very high accuracy of these solutions made them suitable to use as the reference solution against which to compare all of the solutions**

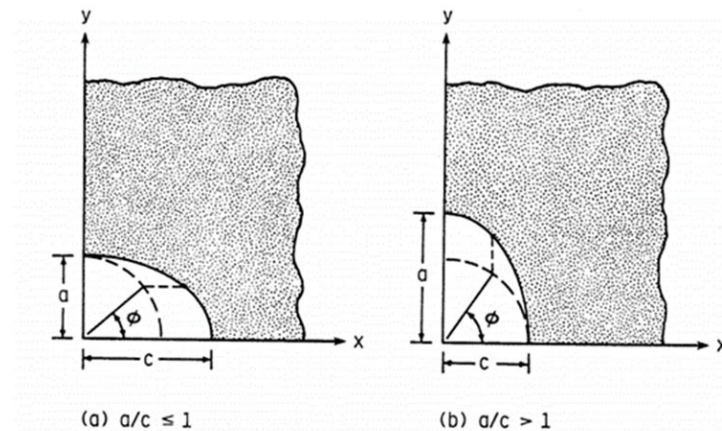
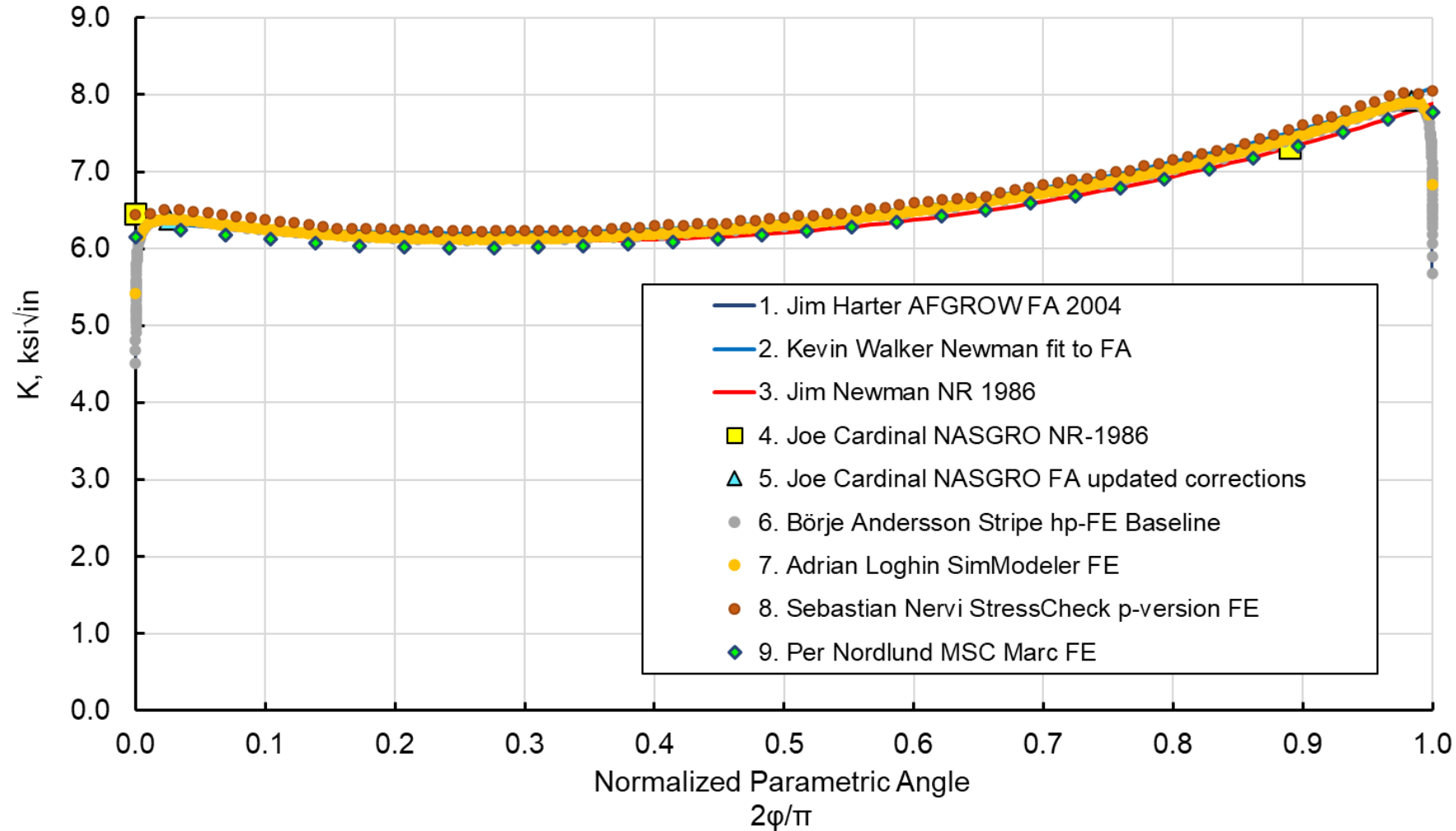
- Submission 7 used SimModeler Crack
- SimModeler Crack is a pre- and post-processor designed for component level finite element-based 3D fatigue crack growth simulations
- SIFs computed via displacement correlation technique with the model solution performed in ANSYS
- Mesh refinement with about 300 elements along the crack front for all cases
- Average relative difference from the Submission 6 reference solution was 0.23%

- Submission 8 used StressCheck p-version FE code
- Modelling based on the approach an everyday practitioner would apply – targeting an estimated maximum error in SIF values of within 2%. This target error is based on sensitivity studies that have shown a variation in SIF of 2% has an effect on the fatigue life of less than 20%.
- SIF was extracted at 200 points along the crack front

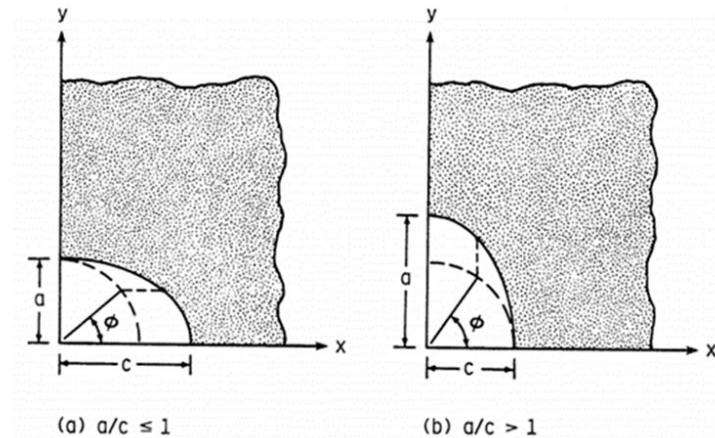
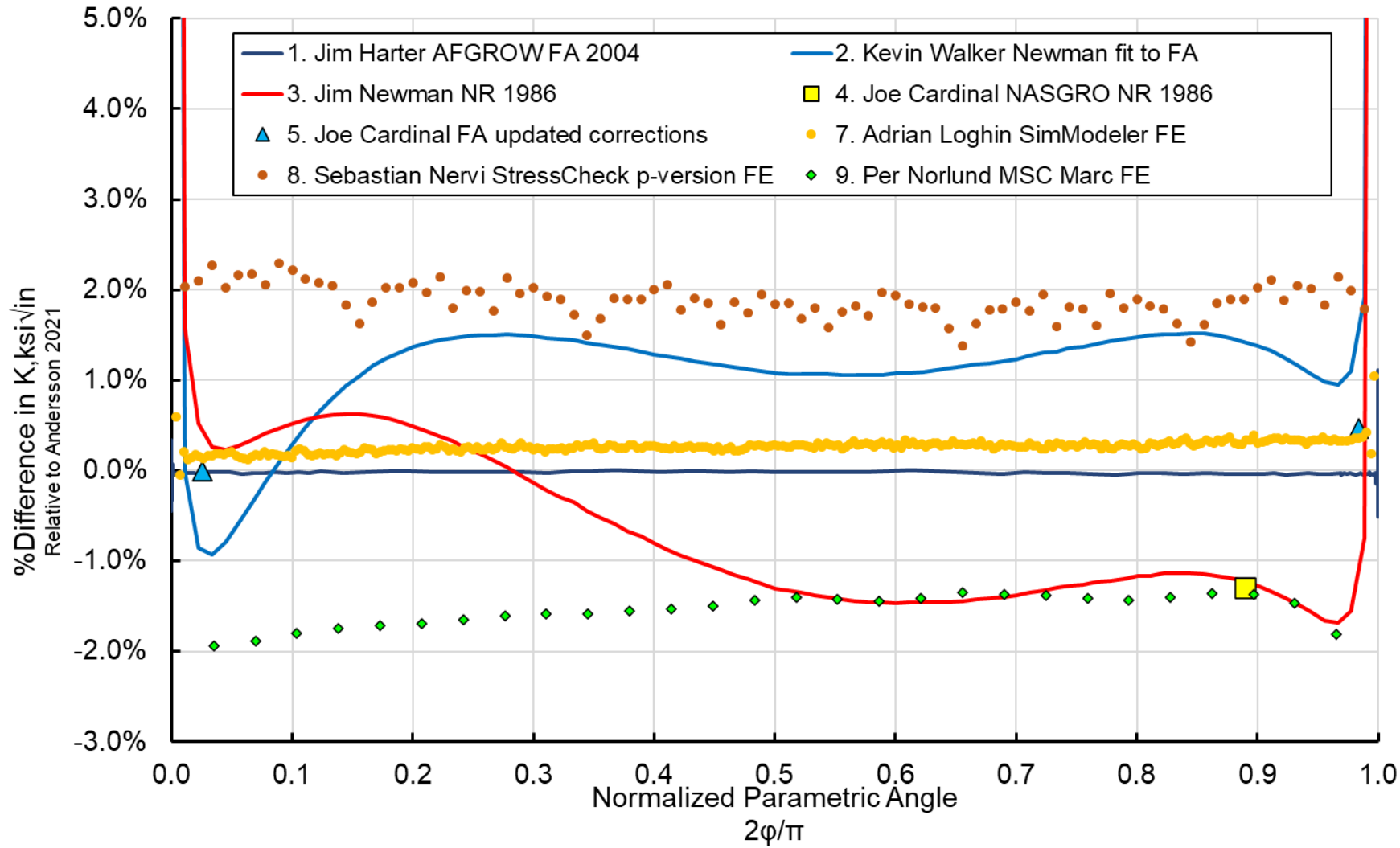
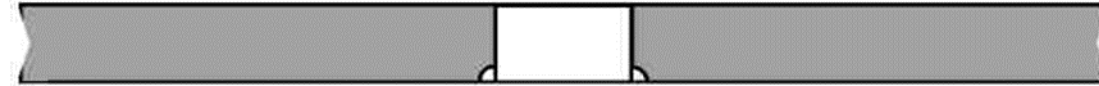
- Submission 9 used Marc from Hexagon MI (former MSC Software) to create 3D FE models
- Marc is a general purpose non-linear solver with special capabilities for crack initiation and crack growth using automatic remeshing
- Automatic mesh generation for a minimum of 30 evaluation points (nodes) along the crack front

Results

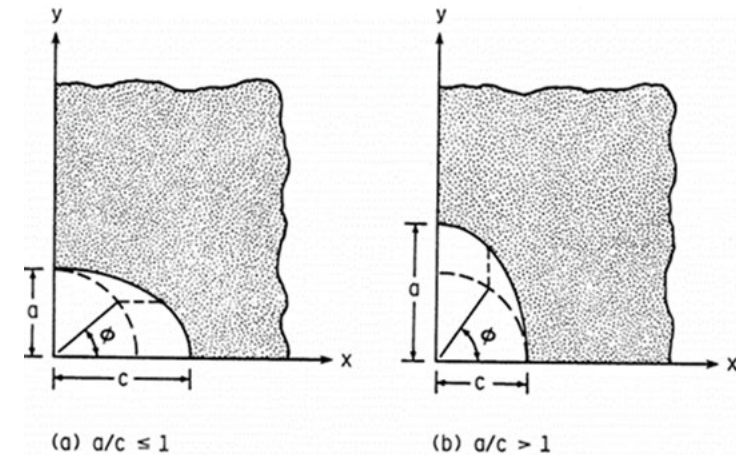
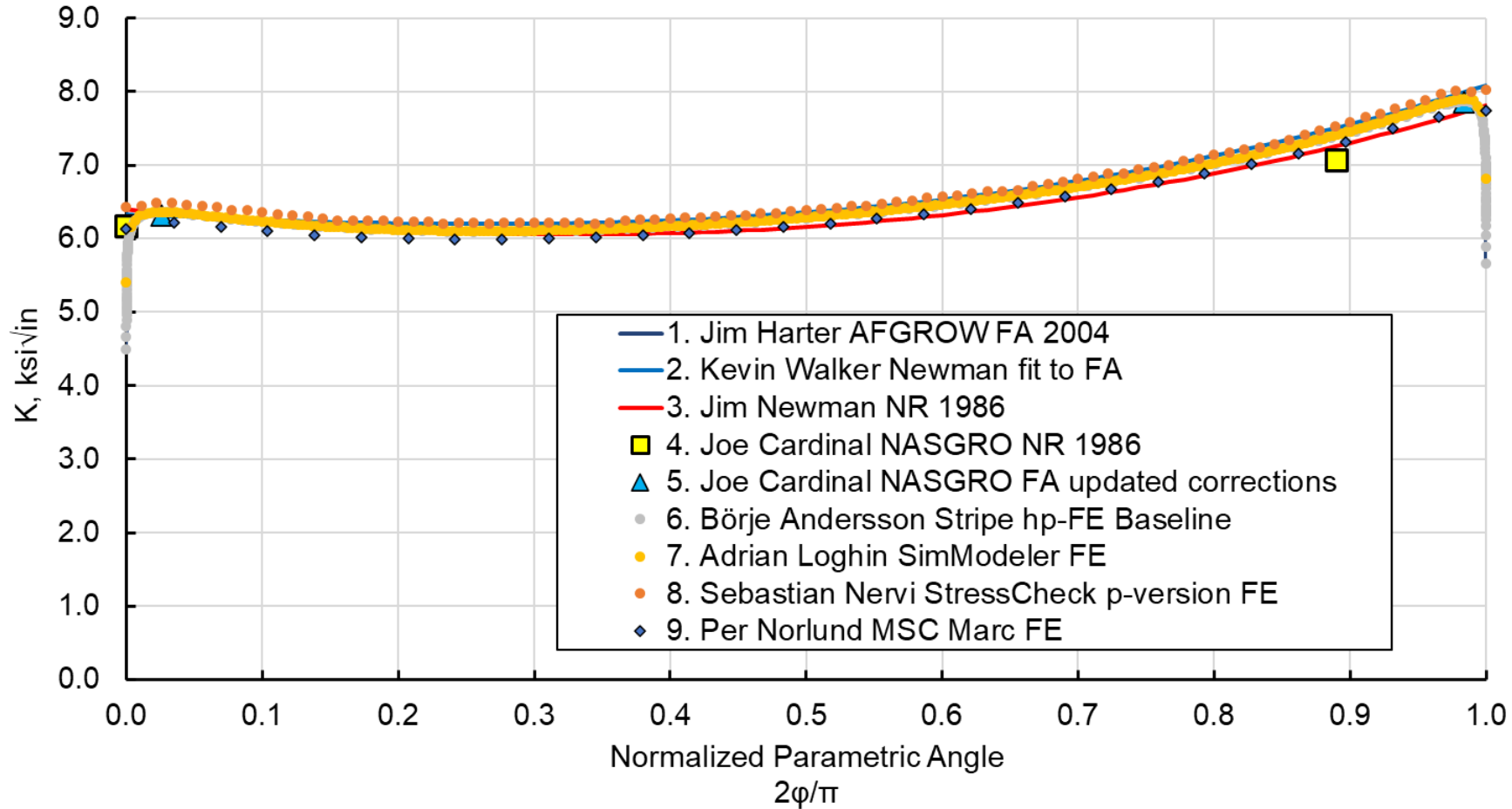
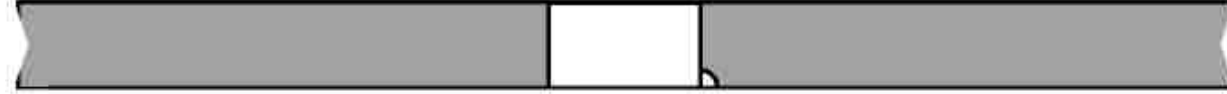
Case 1: Two Symmetric Corner Cracks at a Hole
 $a/c=1, a/t=0.2, r/W=0.0025$



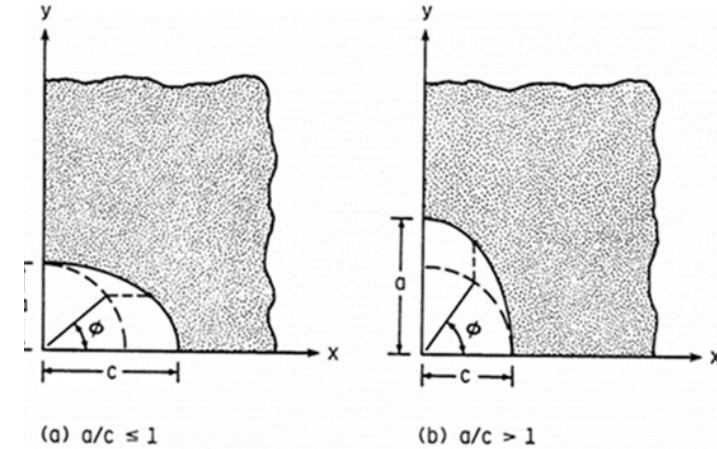
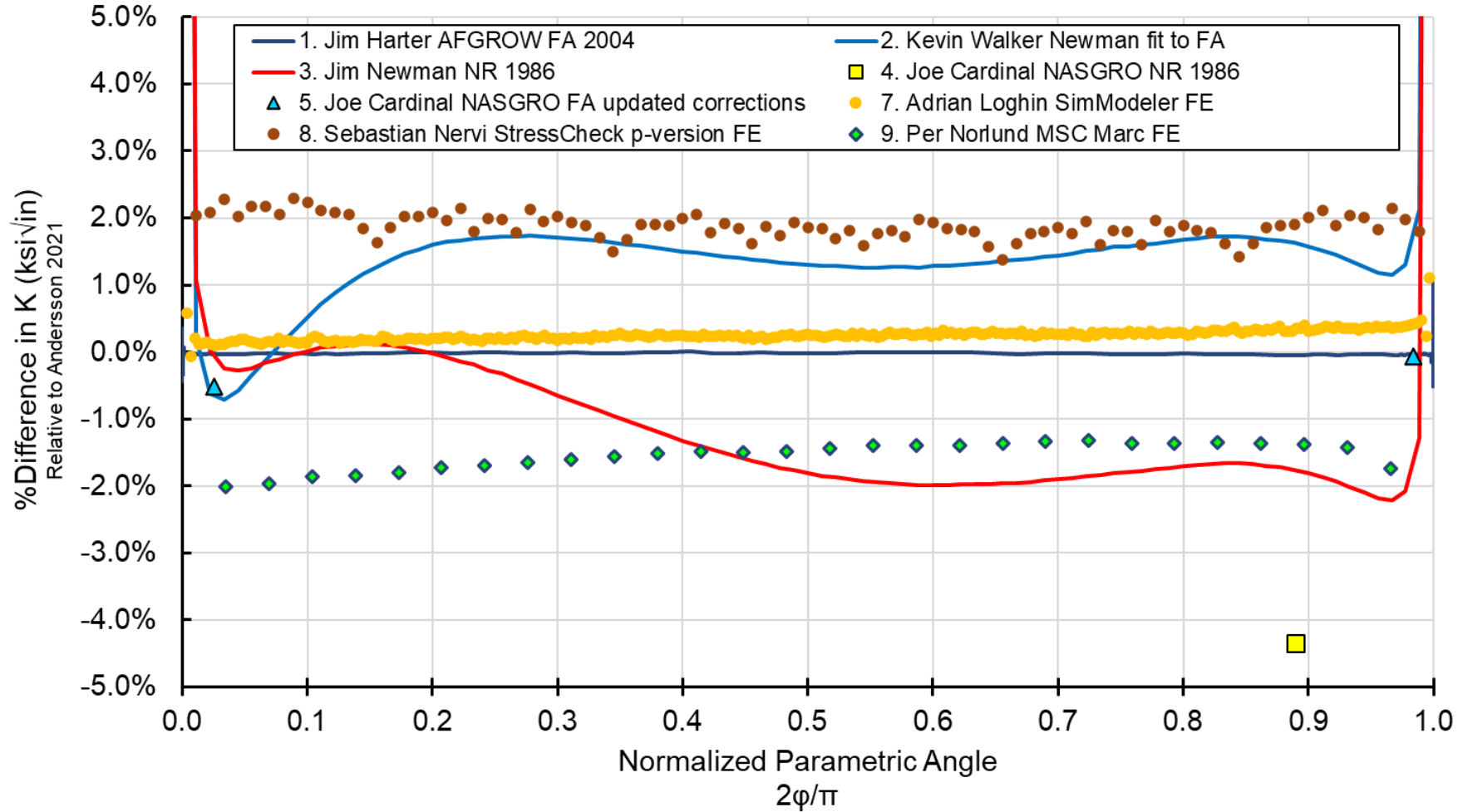
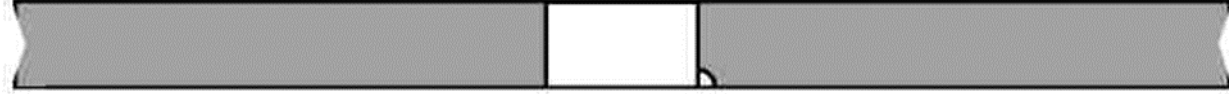
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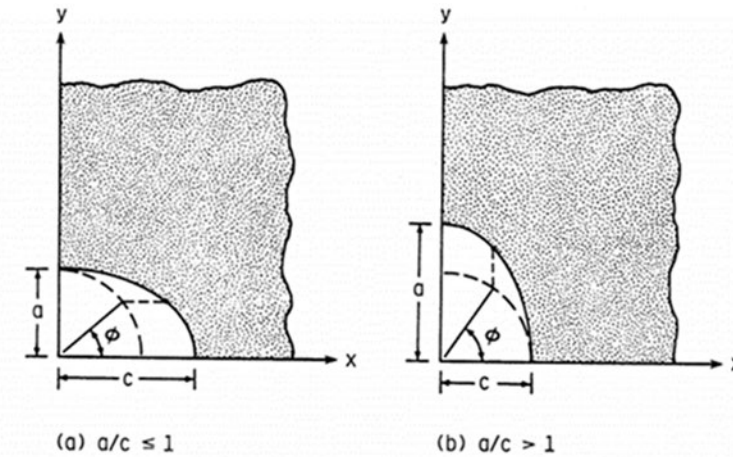
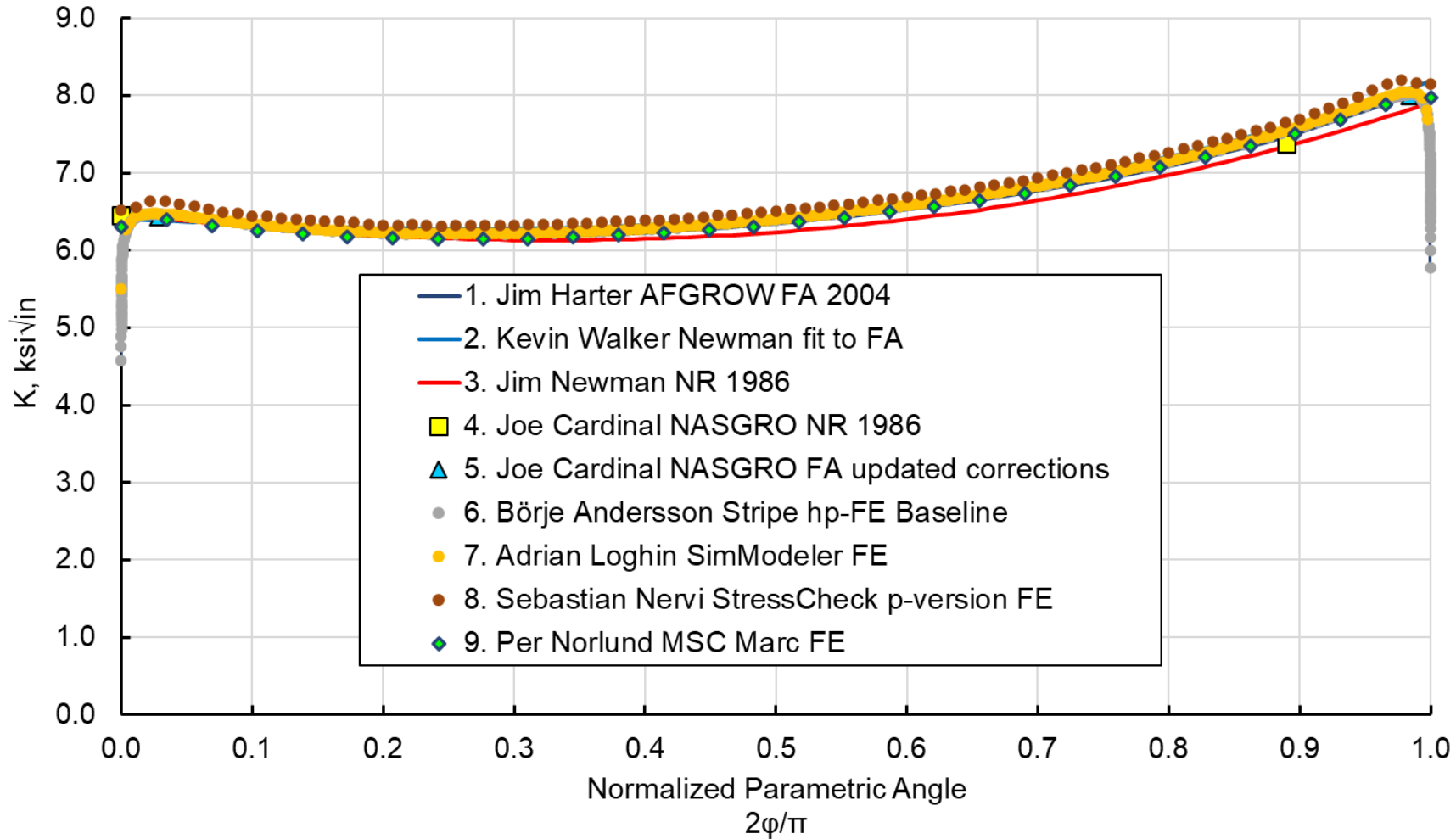
Case 2: Single Corner Crack at a Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.0025$



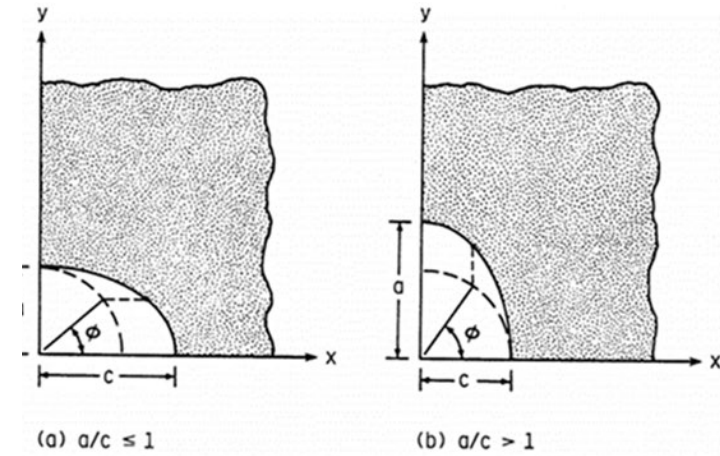
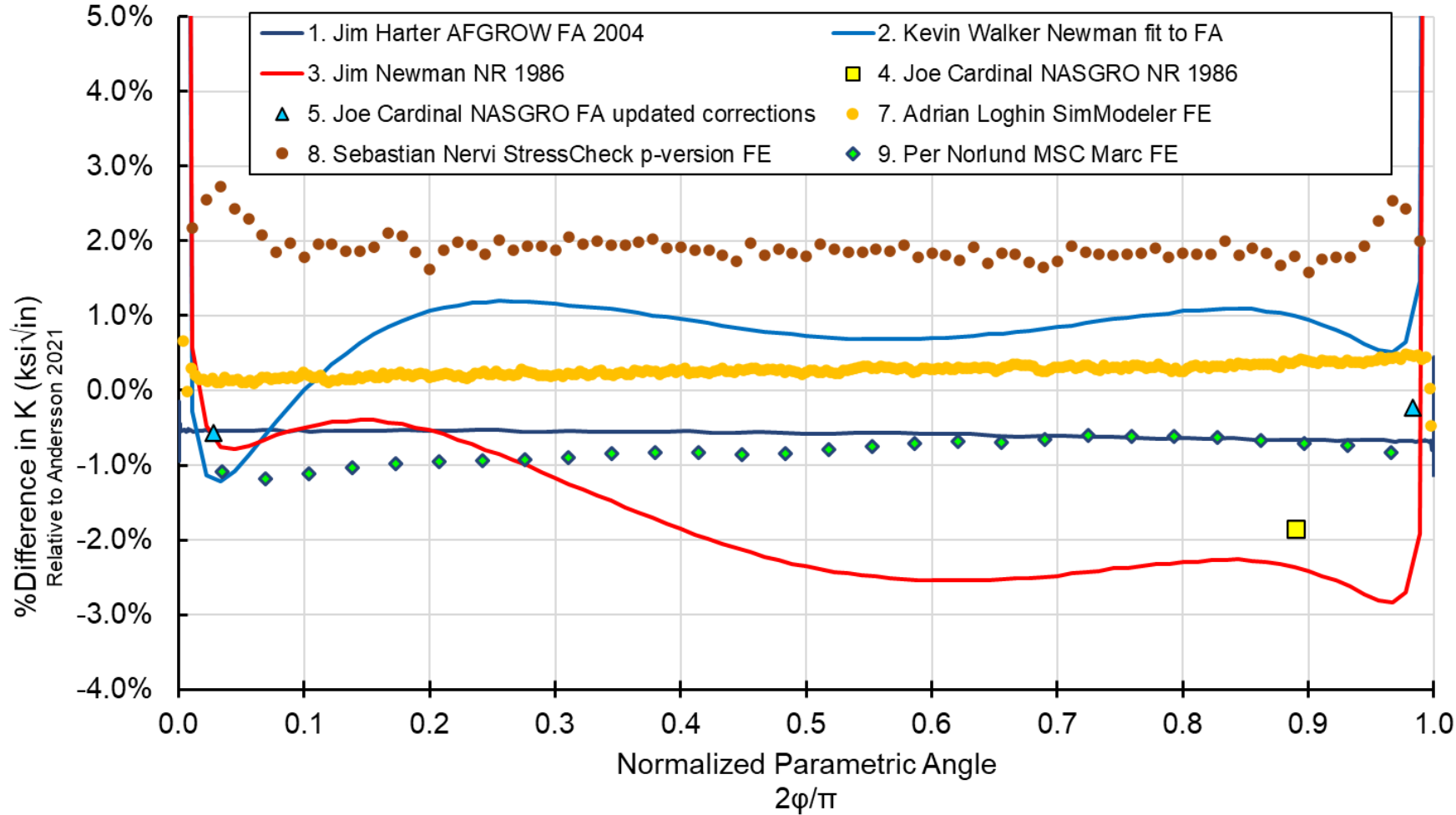
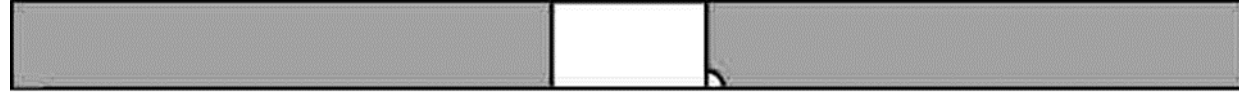
Case 2: Single Corner Crack at a Hole
 $a/c=1, a/t=0.2, r/W=0.0025$



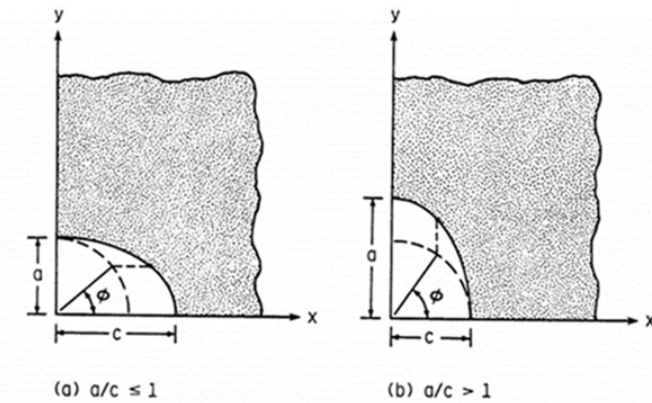
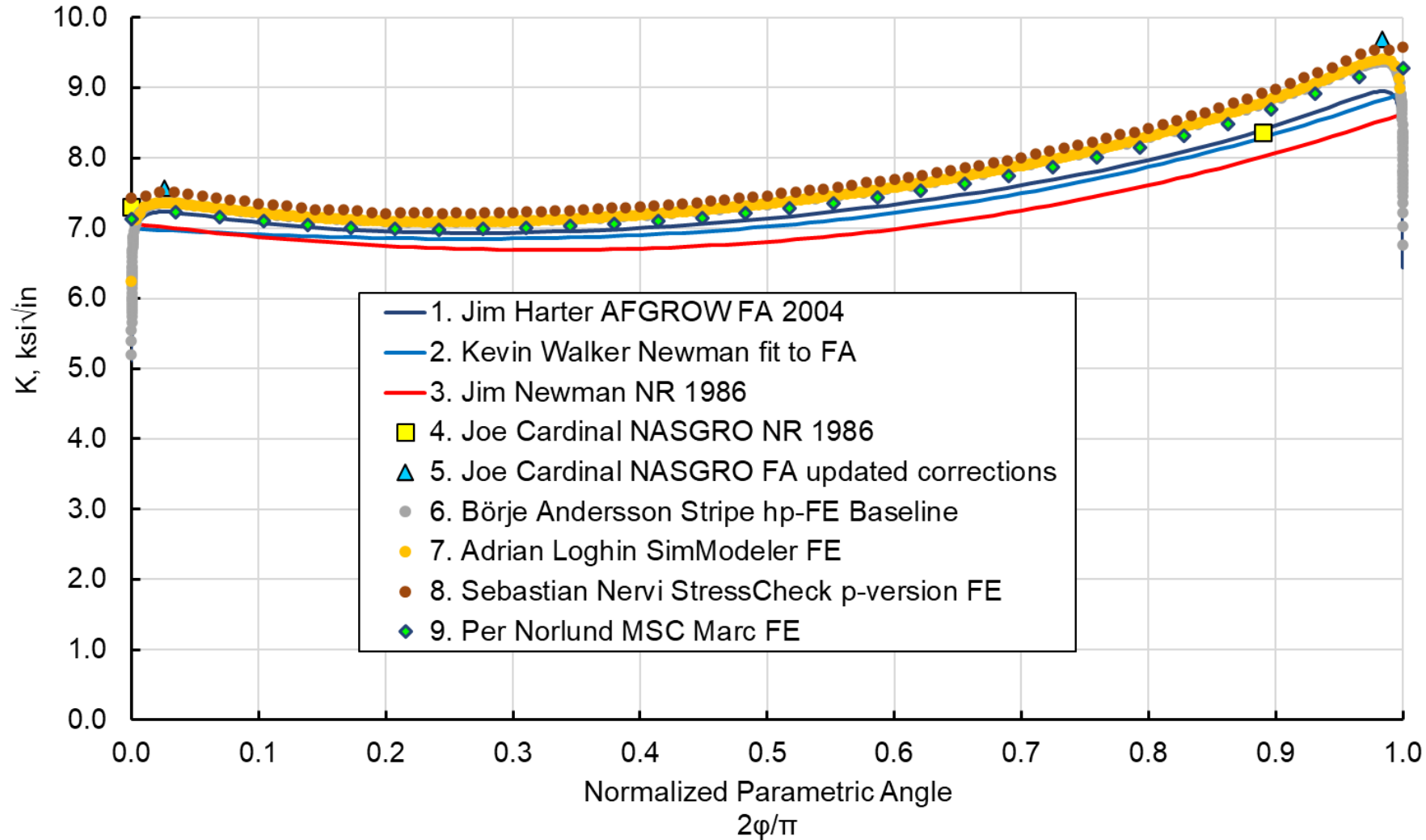
Case 3: Single Corner Crack at a Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.0625$



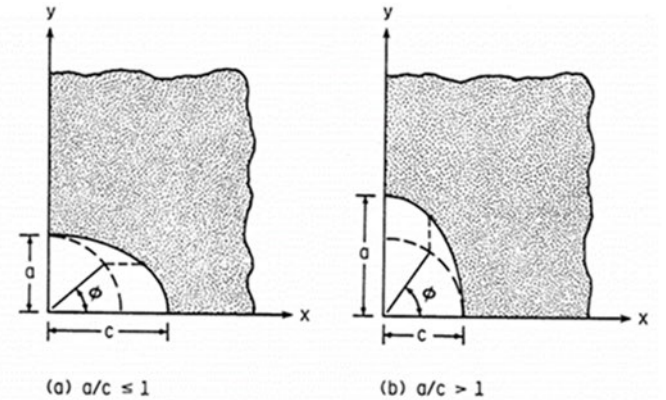
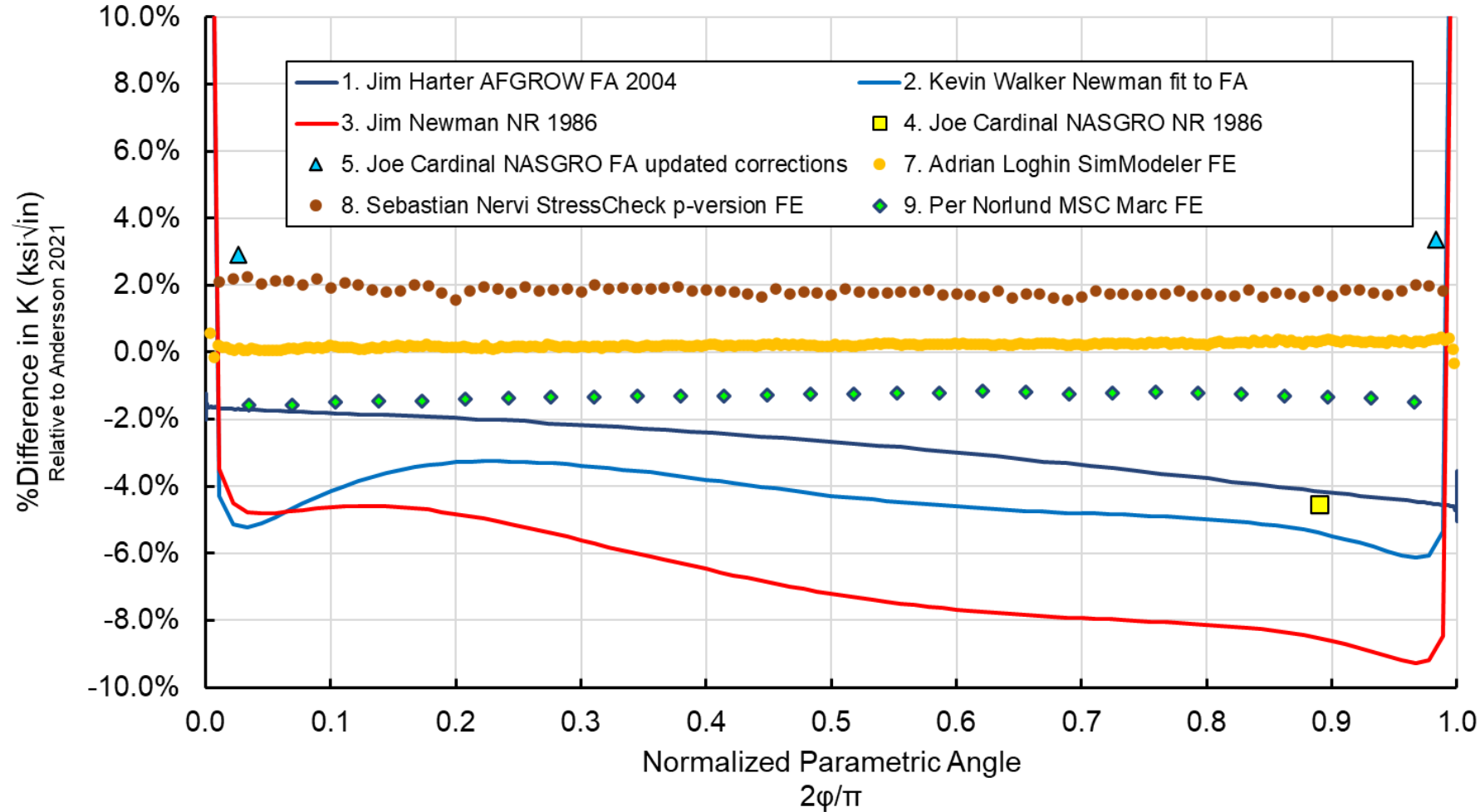
Case 3: Single Corner Crack at a Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.0625$



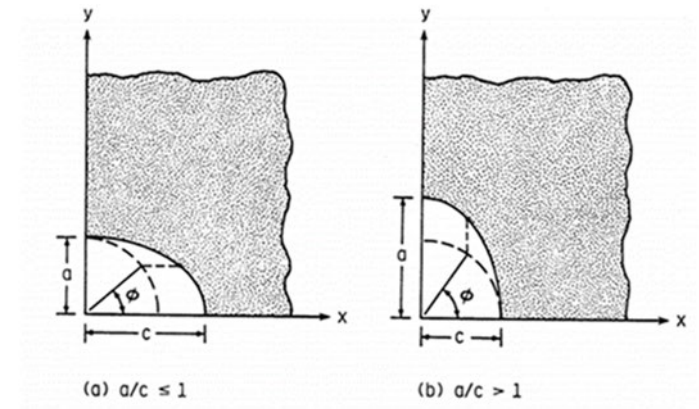
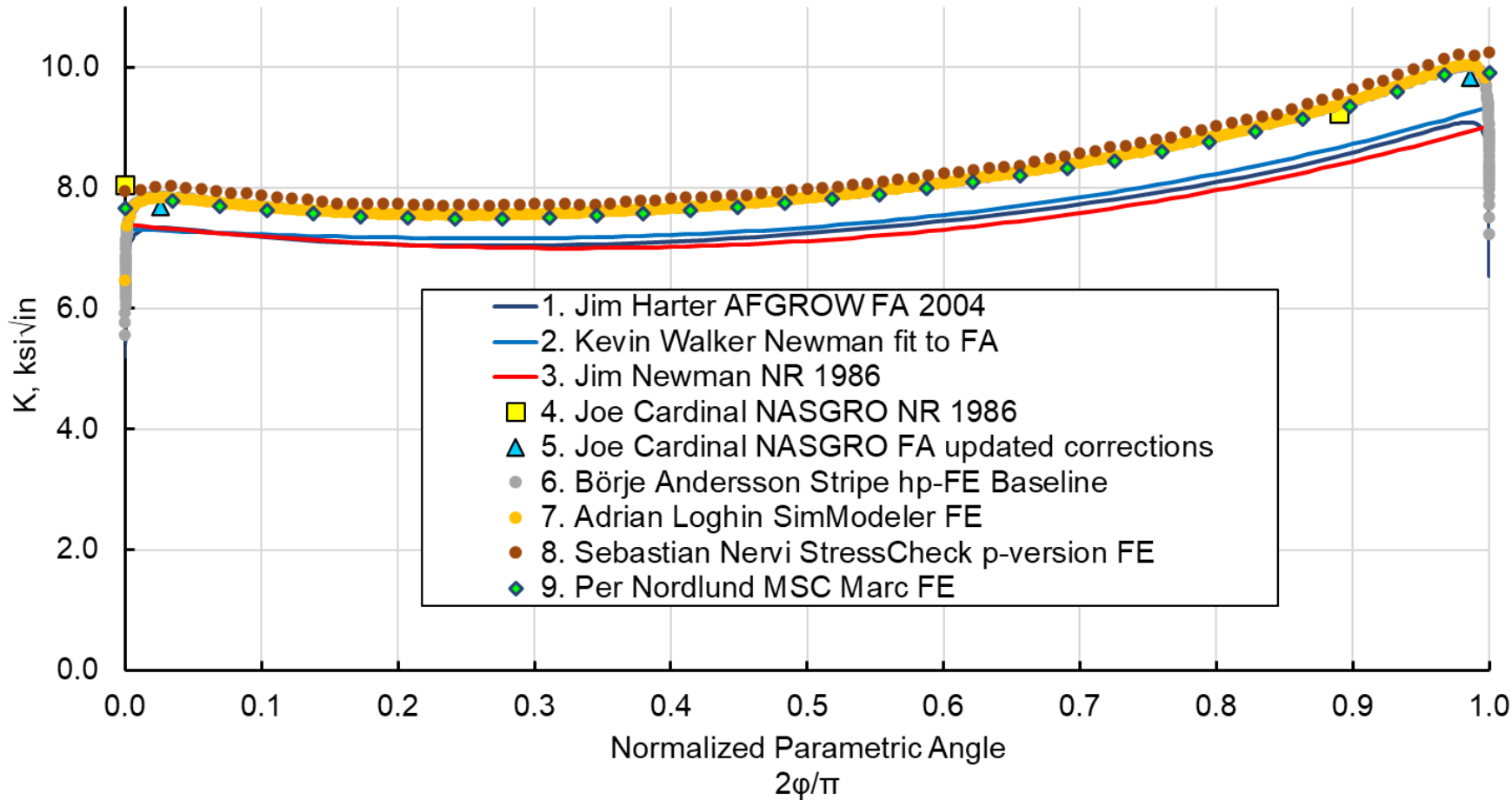
Case 4: Single Corner Crack at an Offset Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.0625$, $e/D=1.20$



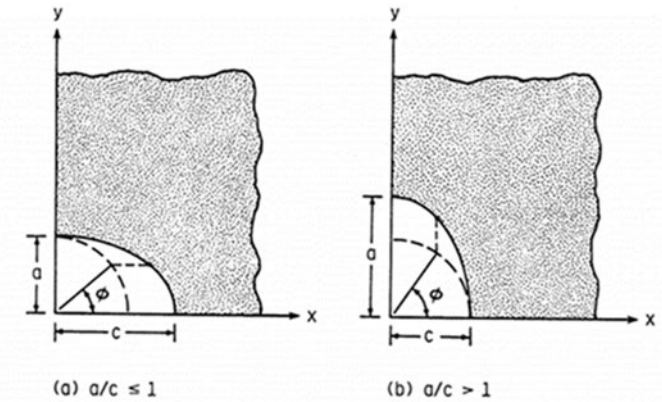
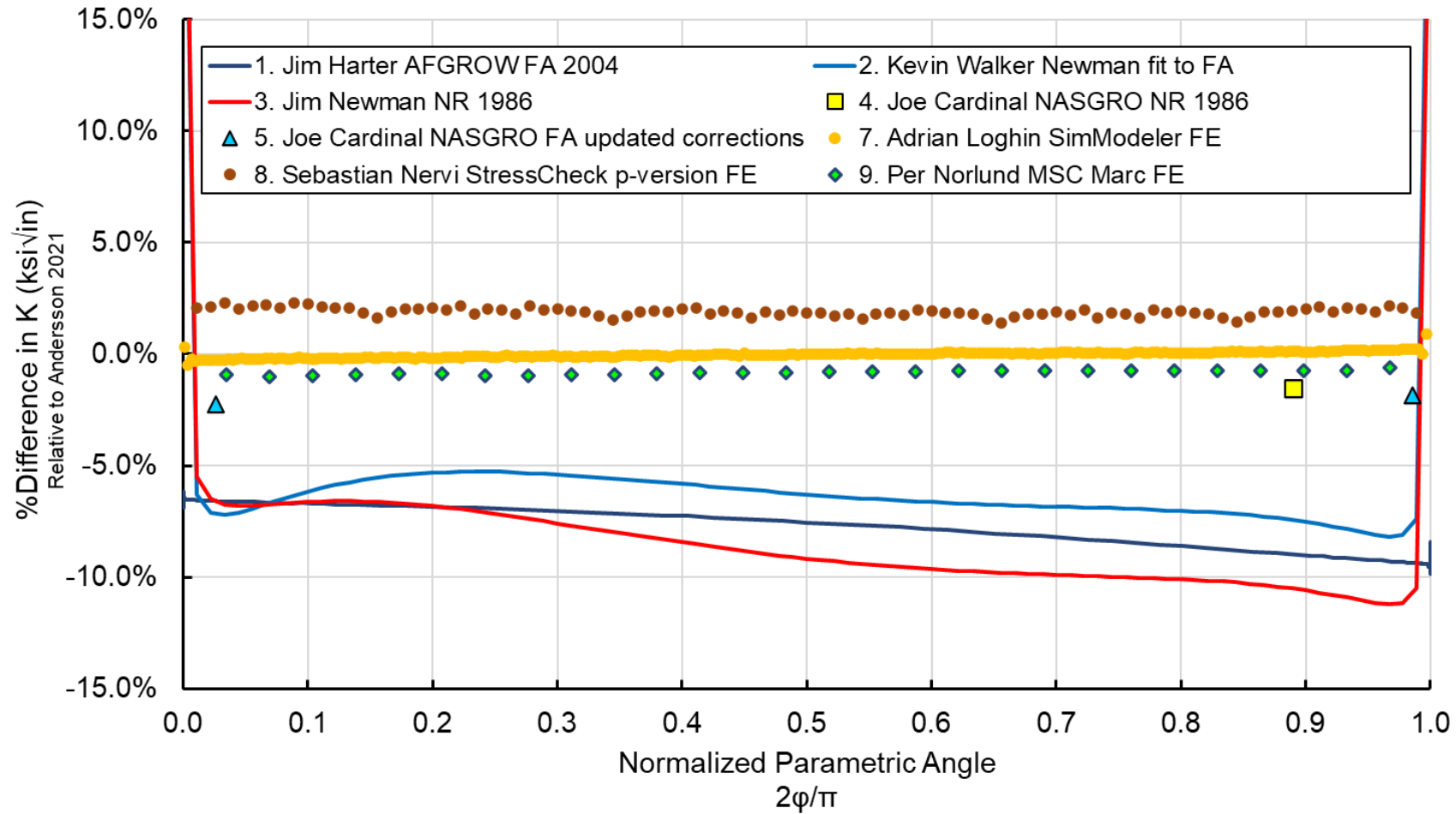
Case 4: Single Corner Crack at an Offset Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.0625$, $e/D=1.20$



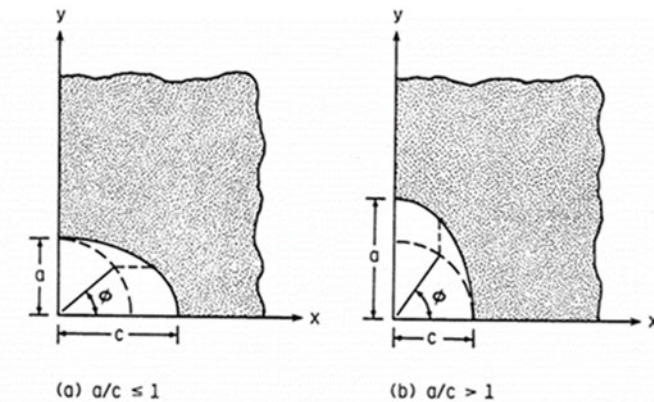
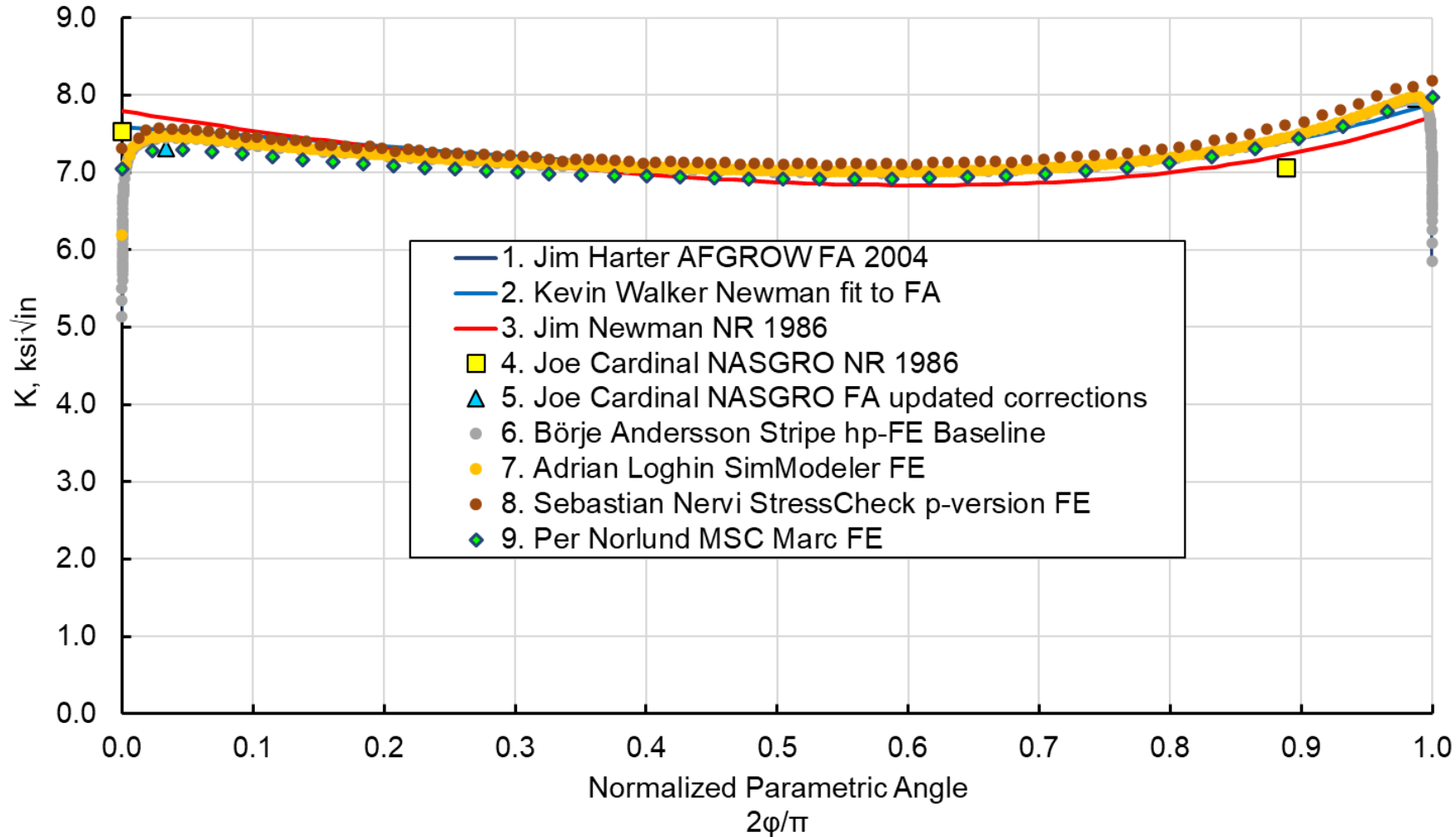
Case 5: Single Corner Crack at Hole
 $a/c=1$, $a/t=0.2$, $r/W=0.2083$



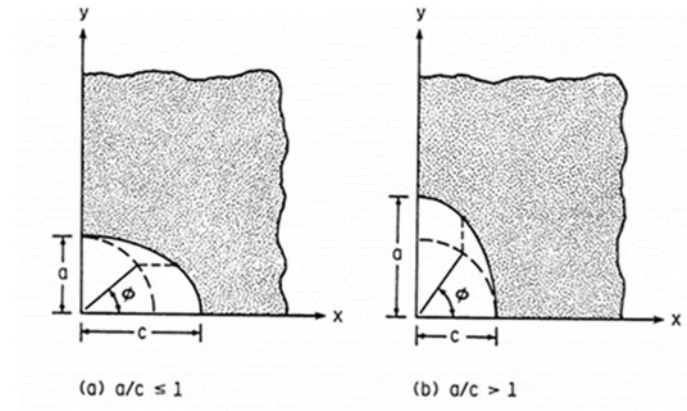
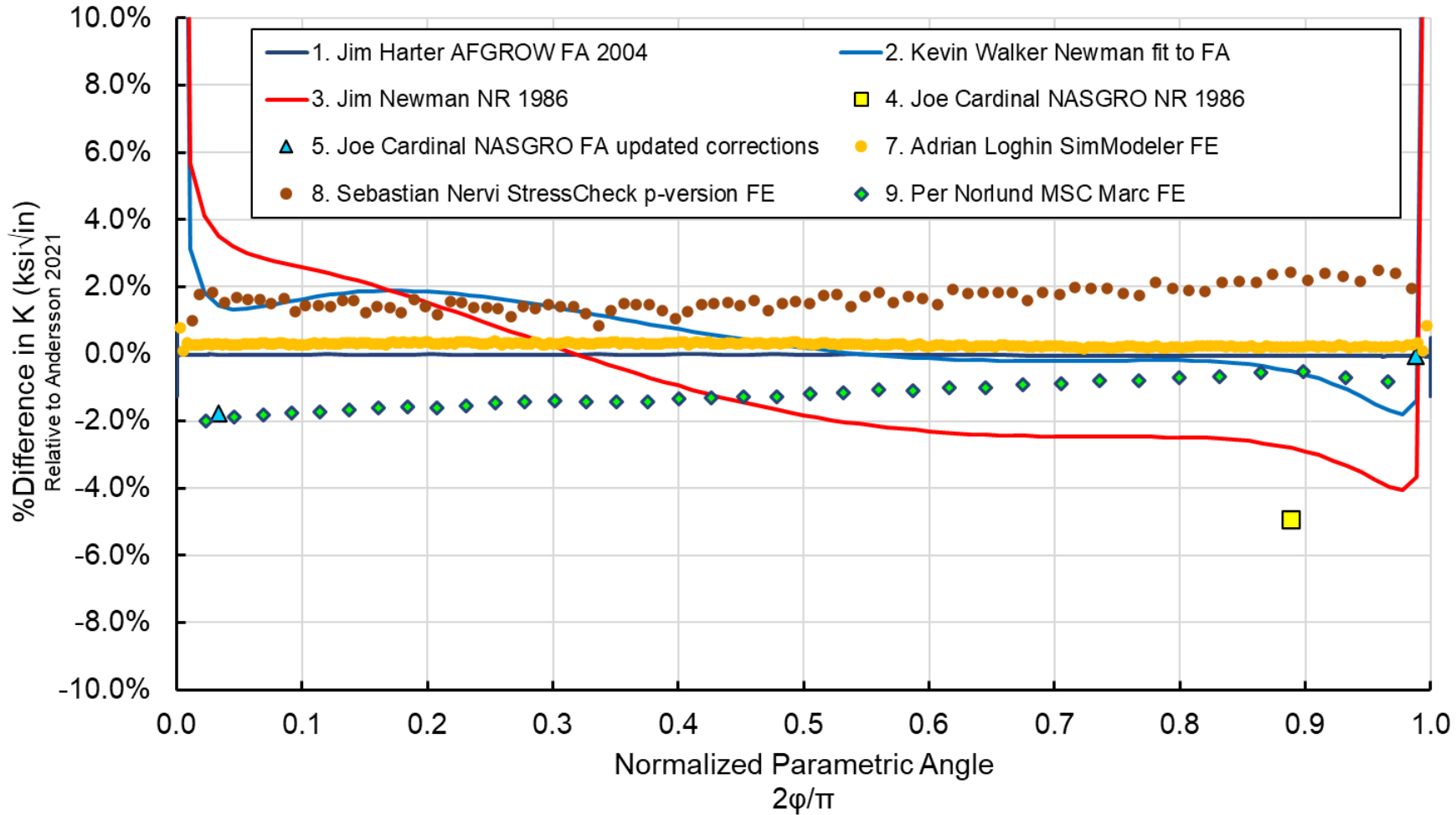
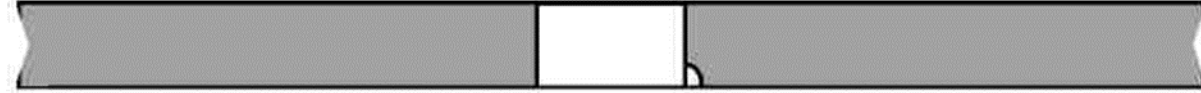
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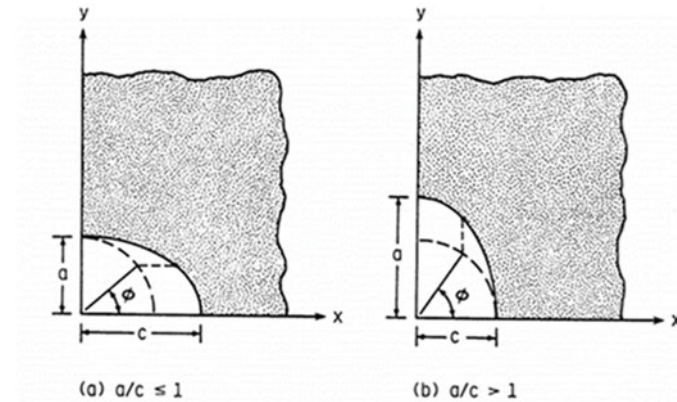
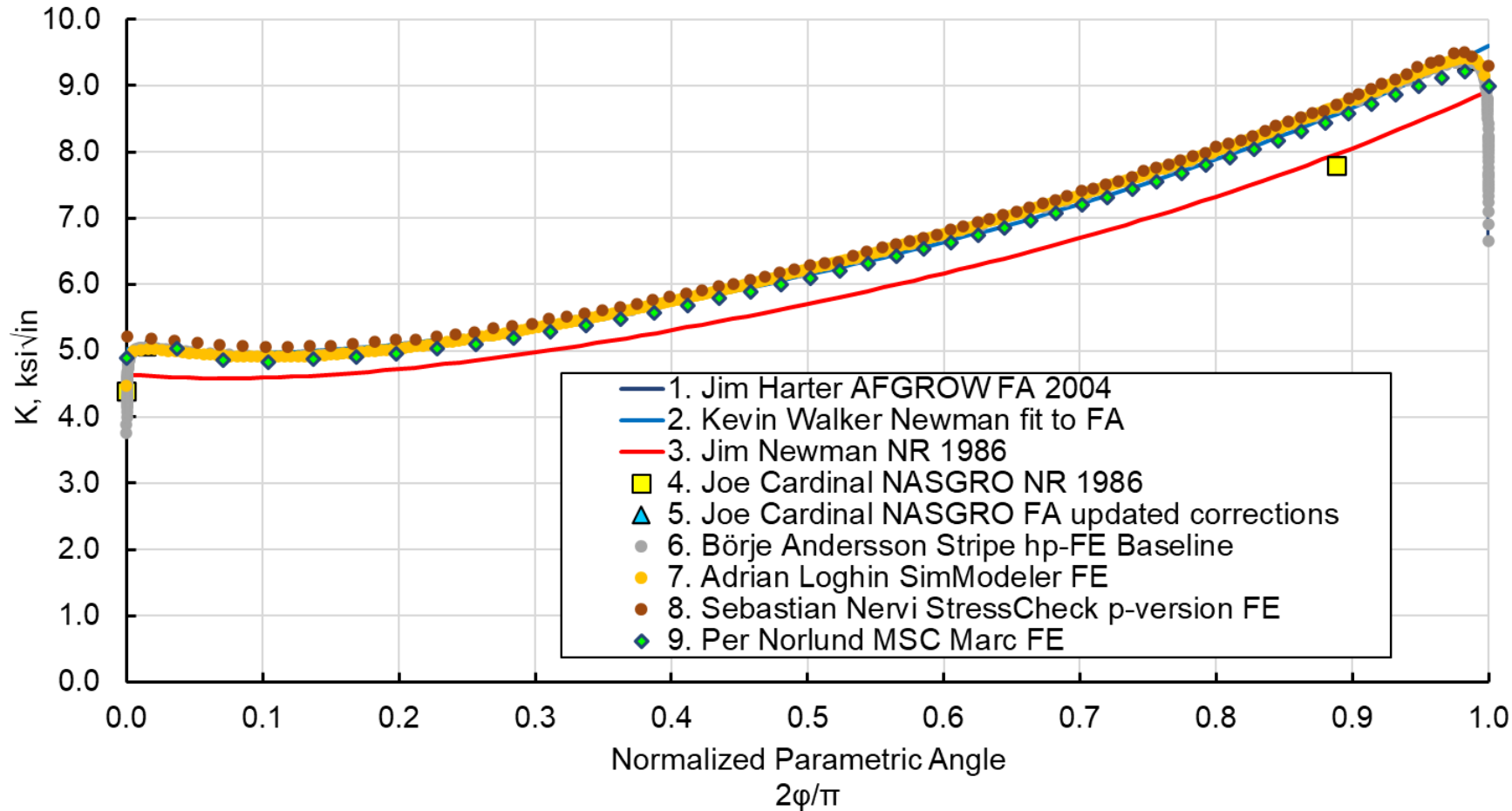
Case 6: Single Corner Crack at Hole
 $a/c=1.5, a/t=0.3, r/W=0.0025$



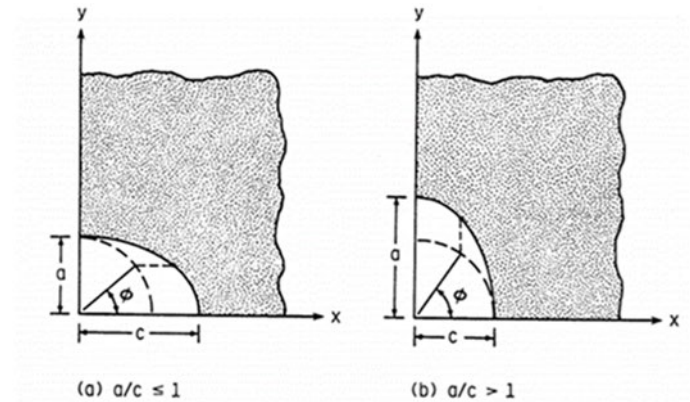
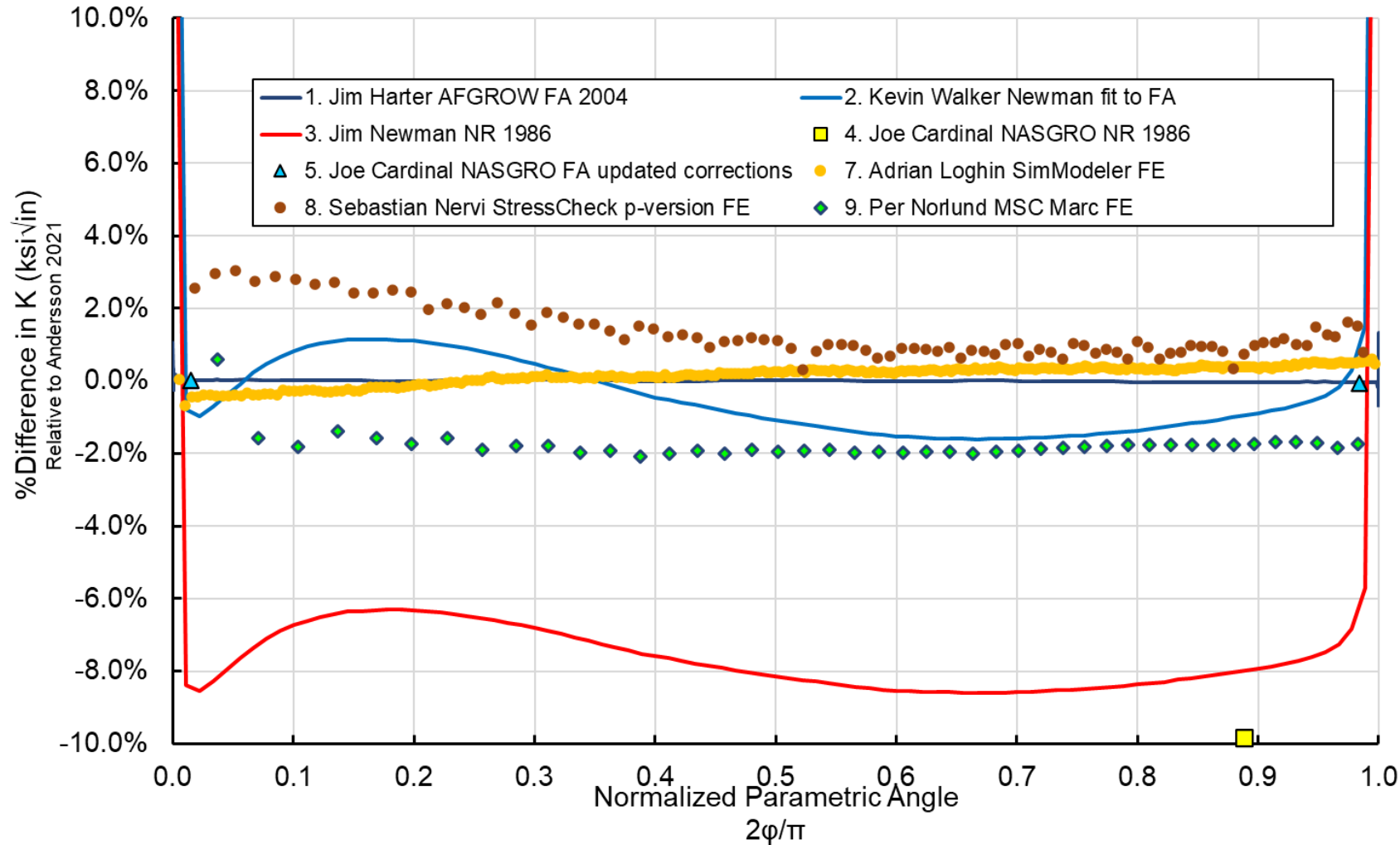
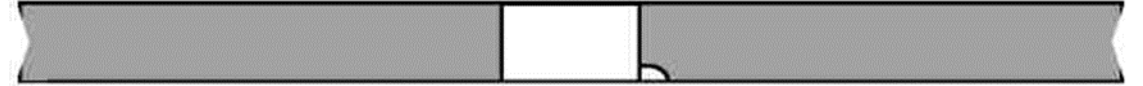
Case 6: Single Corner Crack at a Hole
 $a/c=1.5$, $a/t=0.3$, $r/W=0.0025$



Case 7: Single Corner Crack at a Hole
 $a/c=0.5$, $a/t=0.3$, $r/W=0.0025$



Case 7: Single Corner Crack at a Hole
 $a/c=0.5$, $a/t=0.3$, $r/W=0.0025$

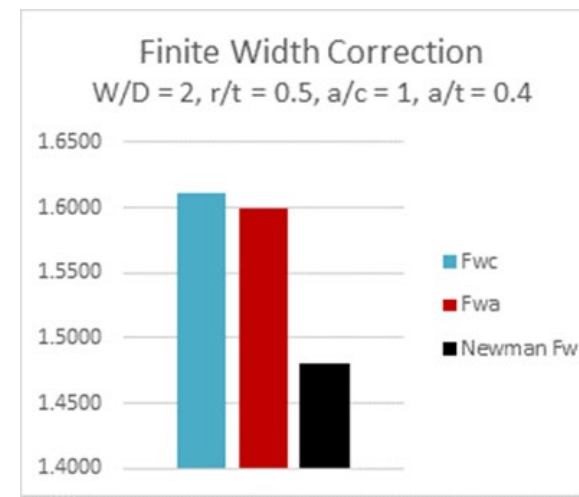
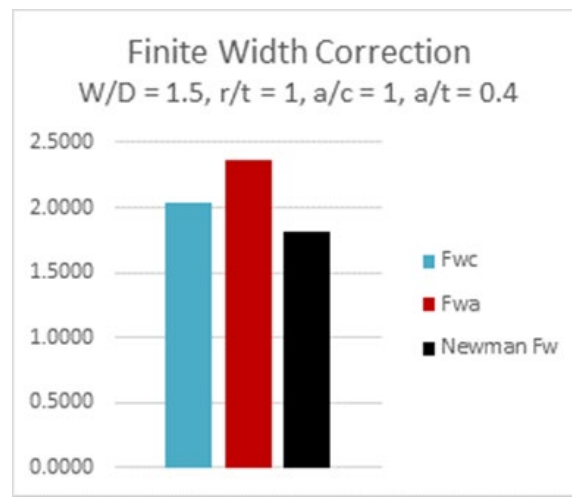


Follow-on investigations

- Case 2 convergence study
- Finite width correction
- StressCheck FEA updated meshing routine

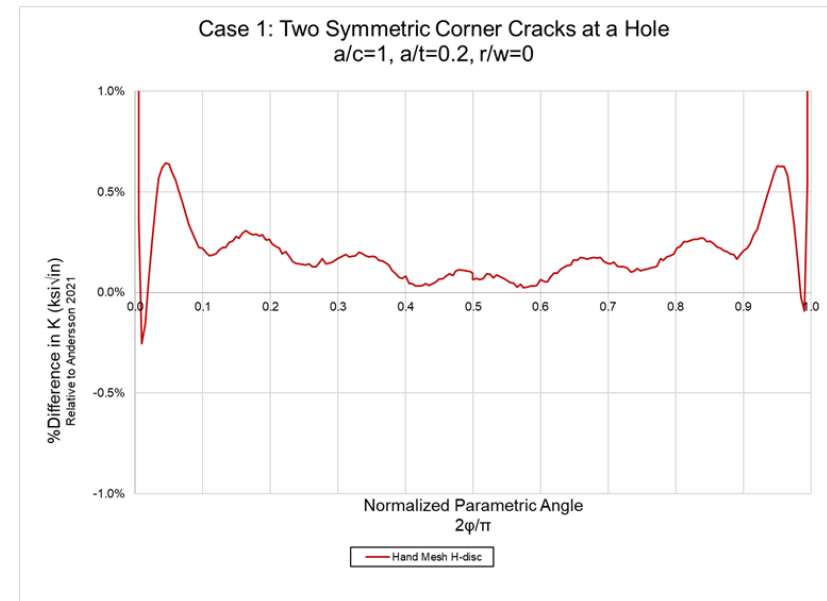
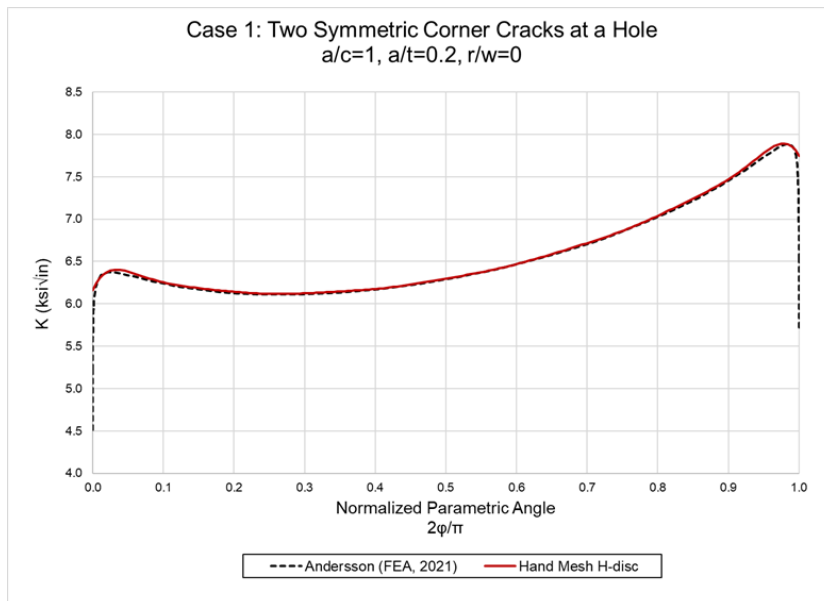
- Fine mesh, $p=8$ solution, STRIPE FE code, and a least-square approximation based analytical expression at the vertices produced a solution accurate to within approximately 0.01% (earlier version was accurate to within 0.03%). This is the reference solution.
- For fine meshes, SimModeler with ANSYS solver produced results within 0.2% relative to the reference solution

- Newman 1986 [7] finite width correction appeared to be inaccurate, particularly for Case 5 (narrow plate 1.2 inch wide)
- Newman 1986 [7] also assumes that the correction applies equally in both the a- and c- directions. This was found to be inaccurate.
- Finite width corrections were determined for several example cases by comparing wide and narrow plate FE solutions (SimModeler) with the same hole and crack geometries
- Improved closed-form finite width correction (function of several parameters including a/c, a/t and r/t) has been developed and will be implemented in AFGROW soon.



StressCheck updated meshing routine

- A second submission was sent targeting a 0.5% tolerance.
- To minimize computational effort an initial coarse parent hand-mesh of hexahedra and pentahedral elements, which was further refined with h-Discretization, was used.
- This meshing approach is utilized primarily for problems of simple topology that are described parametrically and deployed in ESRD's CAE-Handbook.
- Convergence in the estimated error in energy norm (global), and local convergence on SIF, was obtained from the solutions computed using uniform p-extension from p-level 6 to 8.
- The average solution time on a typical Windows 10 engineering laptop (2.9GHz Intel Xeon with 32GB RAM) running solutions for p-levels 6 to 8, including extraction times was less than one minute.



Conclusions

- The Newman-Raju [7] closed-form solutions from 1986 are accurate to within 2% for large width. Even for Case 3, 4 inches wide $W/D=8$, the maximum error is about 3%. But the inaccuracy increases for different aspect ratios (4% for Case 6 $a/c=1.5$, 8% for Case 7 $a/c=0.5$), and also for offset hole (Case 4) with max error 9%, and narrow plate (Case 2) with max error 11%.
- Submission 2 used a Newman fit to the Fawaz-Andersson solutions from 2004, and an improved single crack correction, resulting in accuracy to within 2% for all except for Case 4 with the offset hole (0.6 inch offset) – 6% maximum error, and Case 5 narrow 1.2 inch wide – 8% maximum error.
- Improved and updated finite width corrections are needed
- Highly accurate finite element solutions can be developed depending on the mesh refinement and element type. These have been demonstrated for a range of FE codes.
- A technical report [23] has been written and journal paper providing all the details has been prepared for the journal Engineering Fracture Mechanics (EFM).

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Questions?