ENGINEERED RESIDUAL STRESS IMPLEMENTATION

Cross-Comparisons of Stress Intensity Factors from Various Sources

The Pathway to Improved SIF Solutions

ERSI FCG Analysis + Test Committee Meeting

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  - Dr. Börje Andersson
    - BARE Research
  - Joseph W. Cardinal
    - Staff Engineer, Structural Engineering Department, Southwest Research Institute
  - Jim Harter
    - Senior Consultant, LexTech Inc.
  - Dr. Adrian Loghin
    - Senior Application Engineer, Simmetrix Inc.
  - Dr. Sebastian Nervi
    - Product Manager, Engineering Software Research and Development (ESRD) Inc
  - Dr. Jim Newman
    - Emeritus Professor, Department of Aerospace, Mississippi State University
  - Dr. Per Nordlund
    - MSC Software Corporation
  - Dr. Kevin Walker
    - QinetiQ Australia

Background

- An initial FCG Analysis Methods round robin was completed to quantify the epistemic uncertainties in the prediction of crack growth life, given a fixed set of input data, for baseline and cold expanded (Cx) fastener holes [1,2]
  - Specific input data developed to minimize the effect of random uncertainties but analysts were free to use any means to incorporate the residual stresses into their FCG life prediction
  - The effort was an opportunity to exercise various analytical methods, comparing them to experimental results and uncovering strengths and weaknesses of the various approaches
- During this initial round robin, the prediction sensitivity to the analysis inputs was highlighted with one specific case identifying the influence of error in the Mode I Stress Intensity Factor (K_I) for applied remote loading
  - For several cases, error resulted in no crack growth (ΔK_I lower than $\Delta K_{I,threshold}$)
- As a result of these findings and subsequent discussions amongst the fatigue crack growth community, a follow-on collaborative round robin was established to investigate differences in stress intensity factors readily available in commercially available software like AFGROW and NASGRO

Objectives

- Primary objective of the Stress Intensity Factor (SIF) round robin:

> Evaluate differences between available SIF solutions for a single corner crack at a fastener hole with remote uniform tension loading

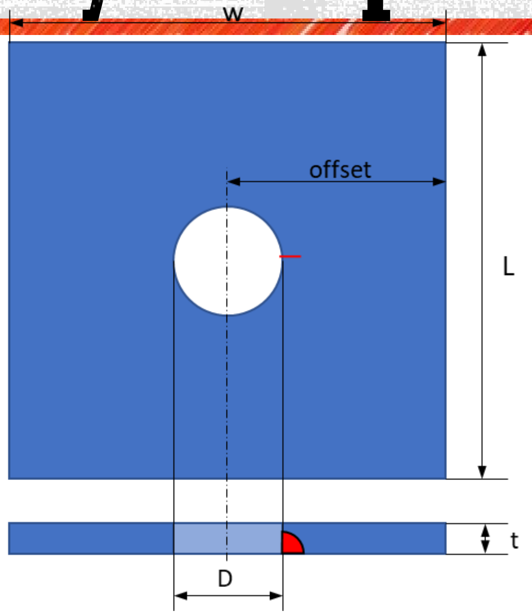
- Evaluations included the root SIF solution and any corrections used to account for any additional corrections applied to the solution
  - Single vs multiple cracks, finite width, and hole offset
- Solutions compared to explicit Finite Element Analysis (FEA) results of each case
- Findings intended to drive improvements to solutions available to the fracture mechanics community

Overview

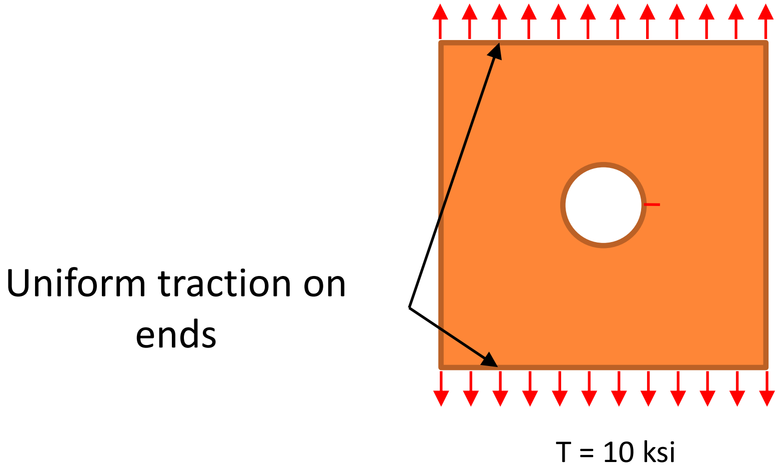
- Seven different cases of corner cracks at a hole were developed and SIF solutions along the crack front were requested from participants
- A building block approach was utilized, with Case 1 representing the root SIF solution available
  - Without any corrections for single cracks, finite width, or hole offset, with a crack geometry aspect ratio (a/c) of 1.0
- Each case added an additional level of complexity with corrections to the root solution as well as variations in the crack aspect ratio

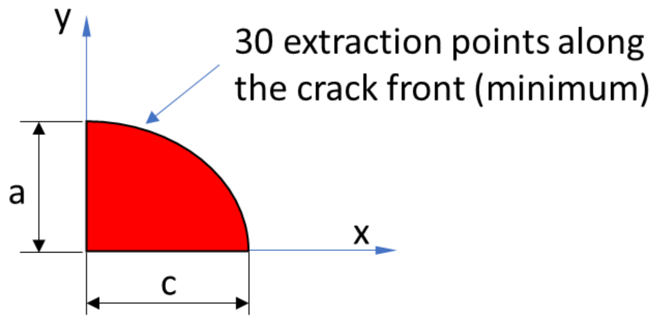
Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

Overview: Analysis Inputs



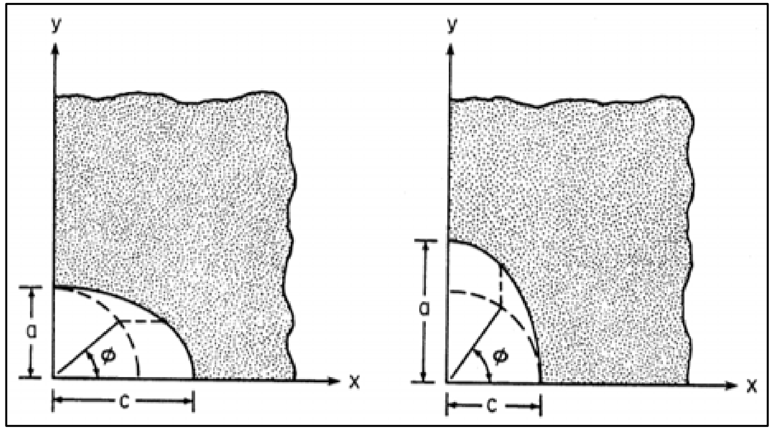
- Geometry
  - Rectangular plate, width W, length L, and thickness t, diameter D
  - Center hole (except Case 4)
- Crack configuration
  - Single corner crack at the hole
  - Except Case 1 which uses double symmetric corner cracks
- Loading conditions
  - Uniform tension stress of 10 ksi applied at the ends of the coupon
- Material properties:
  - Young's modulus E of 10,400 ksi
  - Poisson's ratio ν of 0.3





Overview: Analysis Inputs

- Participants reported Mode I SIF versus the parametric angle
  - Minimum of 30 SIF extraction points along the crack front
- For finite plate configurations (Cases 3-5), L = 3W
- All cases considered a/c = 1 except:
  - Case 6, which considered a/c = 1.5
  - Case 7, which considered a/c = 0.5



Case #	Surface Crack Length (c) (inches)	Bore Crack Length (a) (inch)	a/c	Crack Configuration	Width (inch)	Thickness (inch)	a/t	Hole Diameter (inch)	W/D	r/t	r/W	Offset (inch)	Loading	Reference Stress (ksi)	Notes
1	0.050	0.050	1.00	Double Symmetric Corner Cracks	100.00	0.25	0.20	0.50	200.00	1.00	0.00	50.00	Tension	10.00	Infinite Plate, Double Crack
2	0.050	0.050	1.00	Single Corner Crack	100.00	0.25	0.20	0.50	200.00	1.00	0.00	50.00	Tension	10.00	Infinite Plate, Single Crack
3	0.050	0.050	1.00	Single Corner Crack	4.00	0.25	0.20	0.50	8.00	1.00	0.06	2.00	Tension	10.00	Finite Plate, Single Crack
4	0.050	0.050	1.00	Single Corner Crack	4.00	0.25	0.20	0.50	8.00	1.00	0.06	0.60	Tension	10.00	Finite Plate, Single Crack, Offset Hole
5	0.050	0.050	1.00	Single Corner Crack	1.20	0.25	0.20	0.50	2.40	1.00	0.21	0.60	Tension	10.00	Narrow Plate, Single Crack
6	0.050	0.075	1.50	Single Corner Crack	100.00	0.25	0.30	0.50	200.00	1.00	0.00	50.00	Tension	10.00	Infinite Plate, Single Crack, a/c=1.5
7	0.100	0.050	0.50	Single Corner Crack	100.00	0.25	0.20	0.50	200.00	1.00	0.00	50.00	Tension	10.00	Infinite Plate, Single Crack, a/c=0.5

Submissions

- Nine submissions were received from eight participants, with solutions utilized by
  - AFGROW
  - NASGRO
  - Newman/Raju
  - Fawaz/Andersson
  - Explicit FEA
- FEA approaches utilized various tools and methods which provides an additional opportunity to evaluate the different FEA approaches and their impact on the accuracy of the SIF
- Seven reference solutions which have relative errors in K_I on the order of 0.03% or less were provided by Andersson (Submission 6), and were utilized as the reference solutions for each case evaluated

Submissions

Summary

Submission #	Title	SIF solution source	Single Corner Crack Correction (Cases 2, 3, 4, 5, 6, 7)	Finite Width Correction (Cases 3, 4, 5)	Offset Hole Correction (Cases 4, 5)
1	Fawaz-Andersson Solutions, AFGROW	Fawaz-Andersson [3] (as implemented in AFGROW Advanced Model)	n/a	Newman correction [5]	Harter correction [6]
2	Newman-Raju Fit to Fawaz-Andersson	Updated equations by Newman [7] based on fit to Fawaz-Andersson solutions [4]	Shah-Newman Correction (2020) [8]	Tada, Paris and Irwin correction [9]	• center hole (conservative option) • Kt match approach
3	Newman-Raju (1986)	1986 Newman-Raju solution [9]	Shah correction	Newman correction [5]	Kt match approach
4	NASGRO (CC04 & CC02): Newman-Raju	1986 Newman-Raju solution [9] (as implemented in NASGRO CC04)	Shah correction (as implemented in NASGRO CC02)	NASGRO CC02 [12]	NASGRO CC02 [12]
5	NASGRO (CC16): Fawaz-Andersson	Fawaz-Andersson solutions [3] (as implemented in NASGRO CC16)	n/a	Modified version [13] of the Newman correction [5]	Harter correction [6] (as implemented in NASGRO CC16)
6	Andersson: FEA (2021)	Explicitly modeled each condition utilizing the STRIPE FE-software for the hp-version of the finite element method			
7	SimModeler Crack: FEA (2021)	Utilized SimModeler Crack to create 3D FEMs and compute Mode I SIFs via displacement correlation technique			
8	StressCheck: FEA (2021)	Utilized StressCheck to create 3D FEMs and compute Mode I SIFs			
9	MSC Marc: FEA (2021)	Utilized MSC Marc to create 3D FEMs and compute Mode I SIFs			

Case #	Configuration
1	Infinite Plate, Double Crack
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5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

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Summary of Results

- The following slides summarize comparisons for the seven cases evaluated
  - For these comparisons, the Mode I SIF is plotted along the crack front as a function of normalized parametric angle
  - Percent difference relative to Submission 6 from Andersson is also presented

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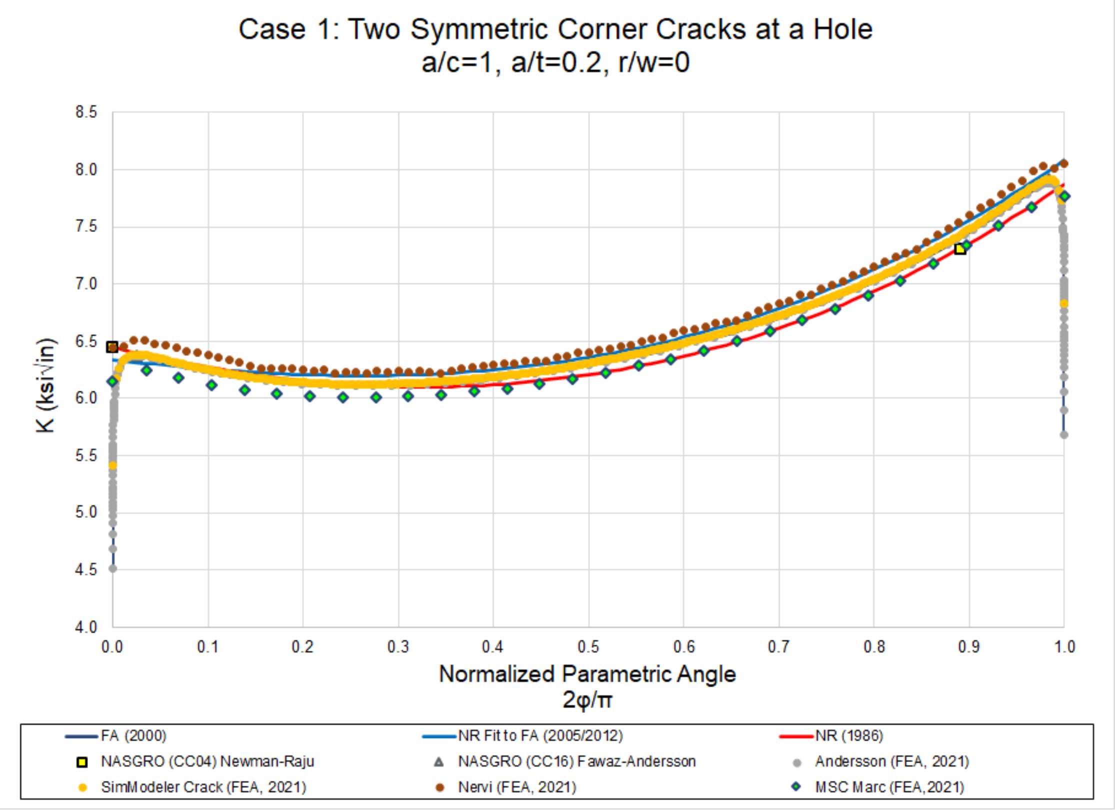
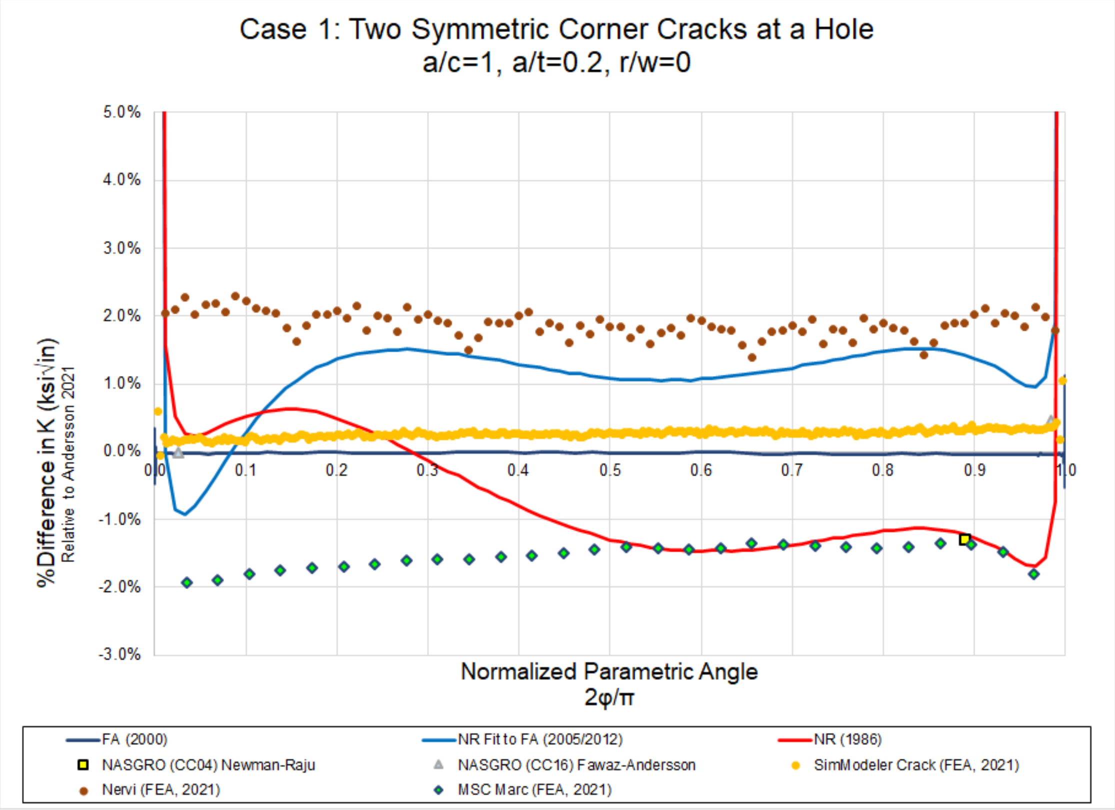
Summary: Case #1

- Case #1: Two Symmetric Corner Cracks at a Hole, Infinite Plate
  - Initial starting point to evaluate the root SIF solutions
    - For this case, single crack, finite width, and hole offset corrections are not utilized
  - Results within ±2% of Andersson submission, except near surface points

Case #	1
Configuration	Infinite Plate, Double Crack
Crack Configuration	Double Symmetric Corner Cracks
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.050
Width	100.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	50.00
a/c	1.00
a/t	0.20
W/D	200.00
r/t	1.00
r/W	0.00





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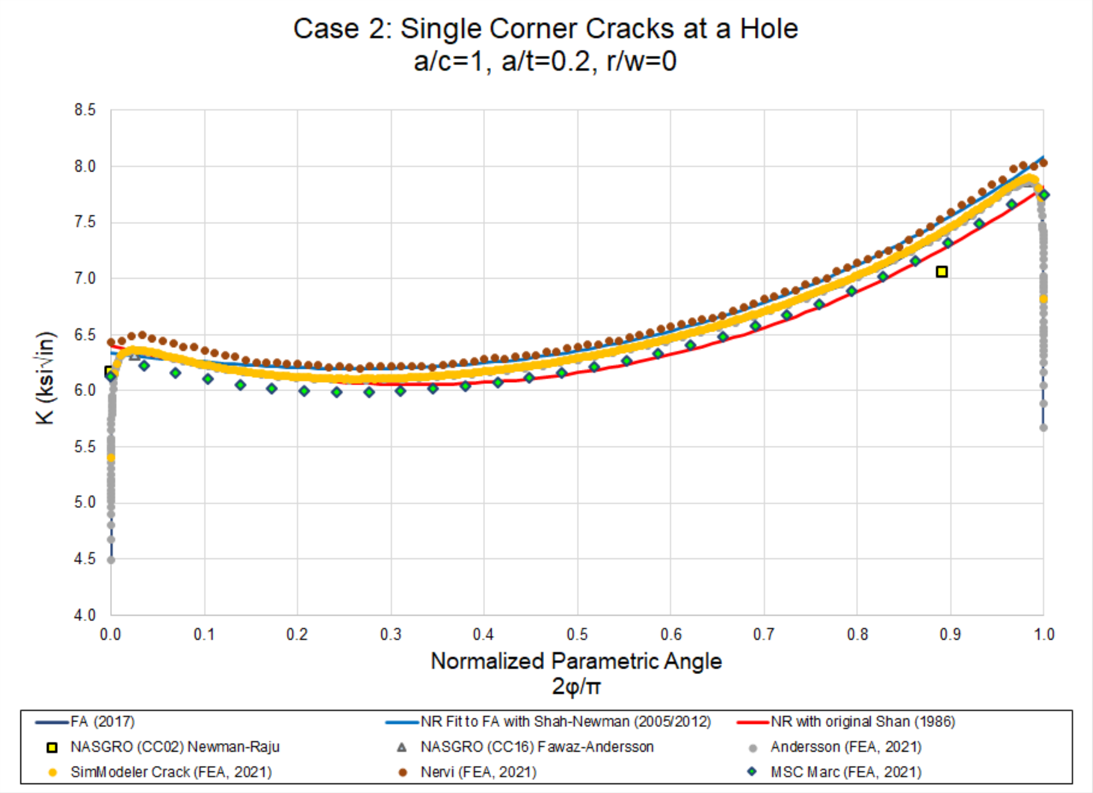
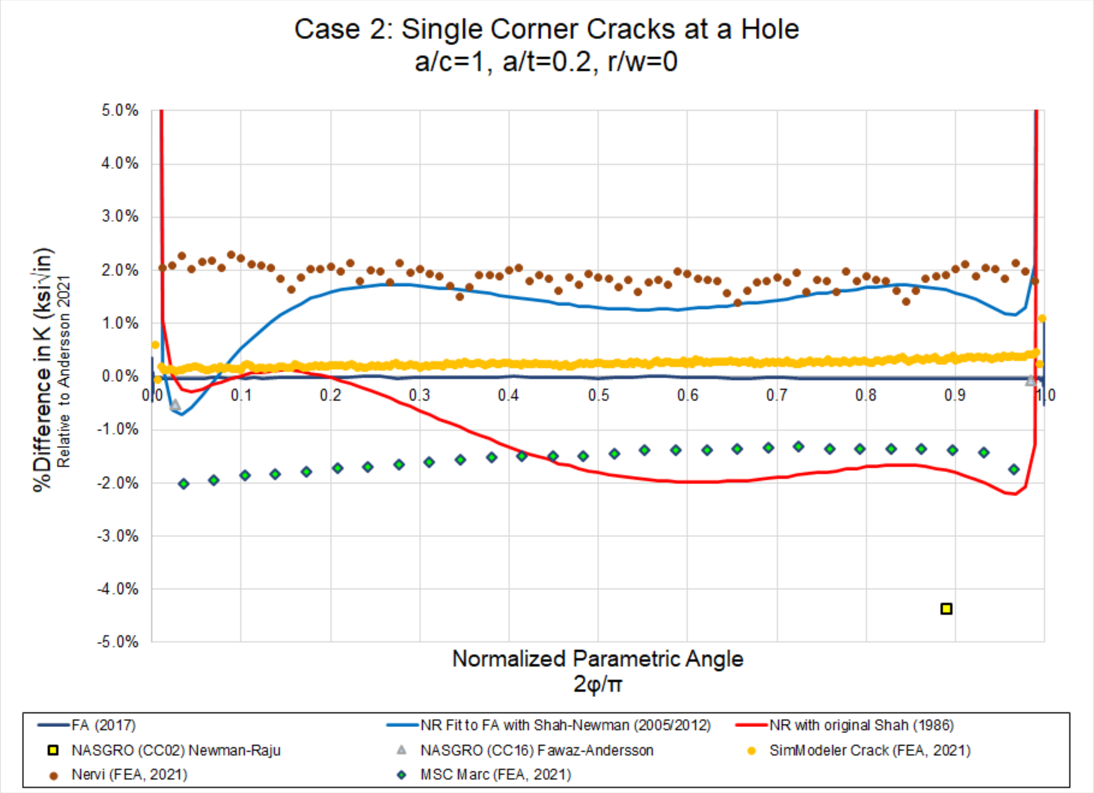
Summary: Case #2

- Case #2: Single Corner Crack at a Hole, Infinite Plate
  - Continuation from Case #1, incorporating effects of a single corner crack
    - Submissions 2-4 utilize Shah or Shah/Newman corrections to adjust from double corner crack to single crack
    - Submissions 1 & 5 utilized single crack modeling in development of root SIF solution
  - Results generally within ±2% of Andersson submission, except near surface points
    - Submission 4 (NASGRO CC02) differences exceeded 4% for point representative of hole bore

Case #	2
Configuration	Infinite Plate, Single Crack
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.050
Width	100.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	50.00
a/c	1.00
a/t	0.20
W/D	200.00
r/t	1.00
r/W	0.00





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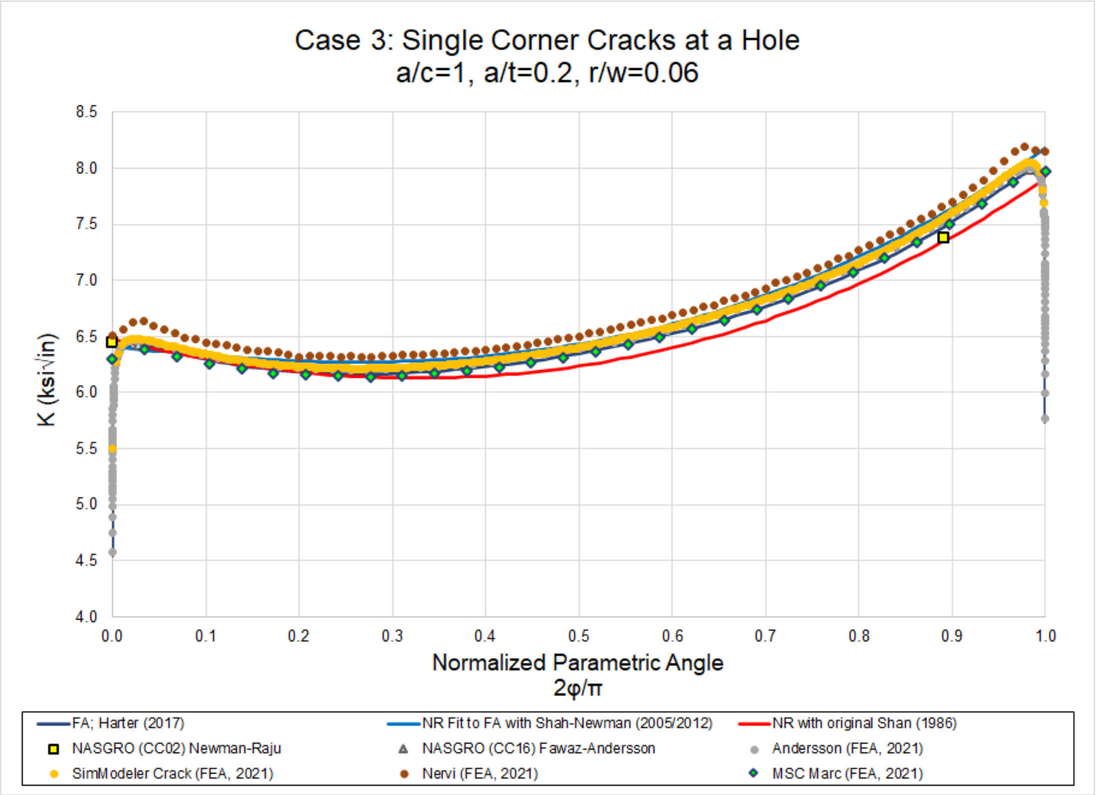
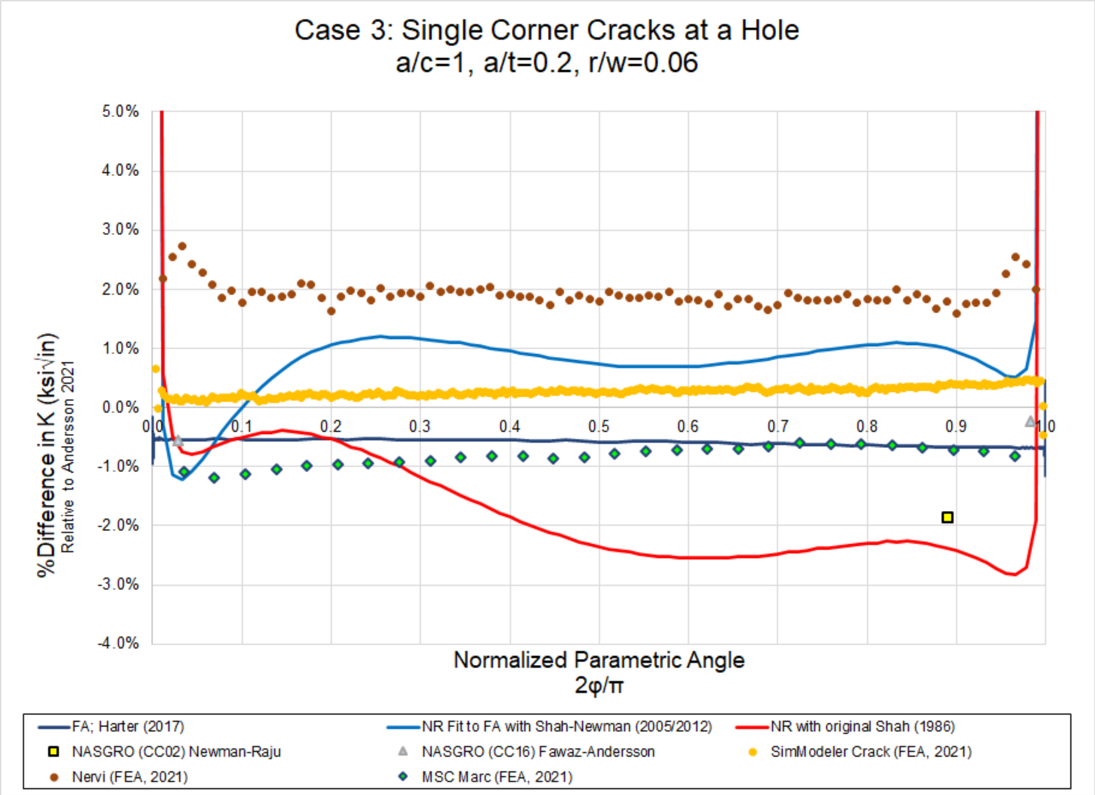
Summary: Case #3

- Case #3: Single Corner Crack at a Hole, Finite Plate
  - Continuation from Cases 1-2, incorporating finite width effects
    - Submissions 1-3 utilized the Newman finite width correction. Submission 4 used the correction from [12] and Submission 5 used the correction from [13]
  - Results generally within ±2% of Andersson submission, except near surface points
    - Submission 3 (Newman-Raju 1986) differences exceeded 2% over a range of 0.4-1.0 normalized parametric angle, representative of crack front near the hole bore

Case #	3
Configuration	Finite Plate, Single Crack
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.050
Width	4.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	2.00
a/c	1.00
a/t	0.20
W/D	8.00
r/t	1.00
r/W	0.06

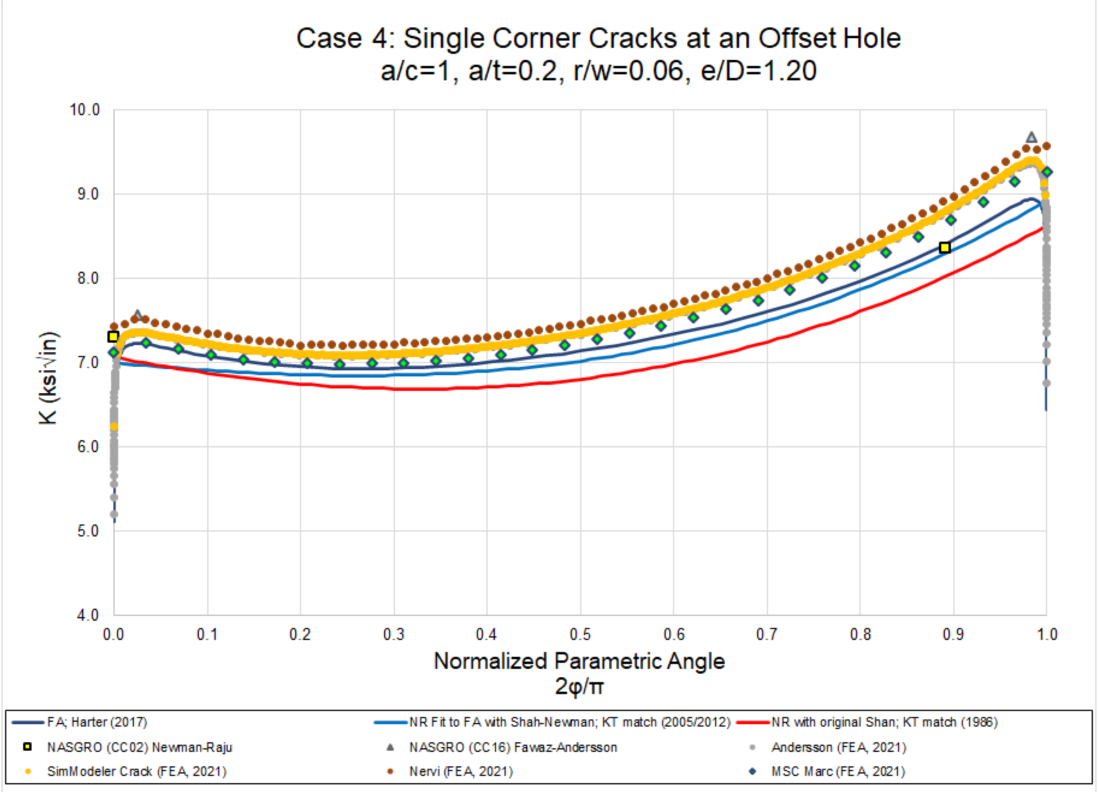


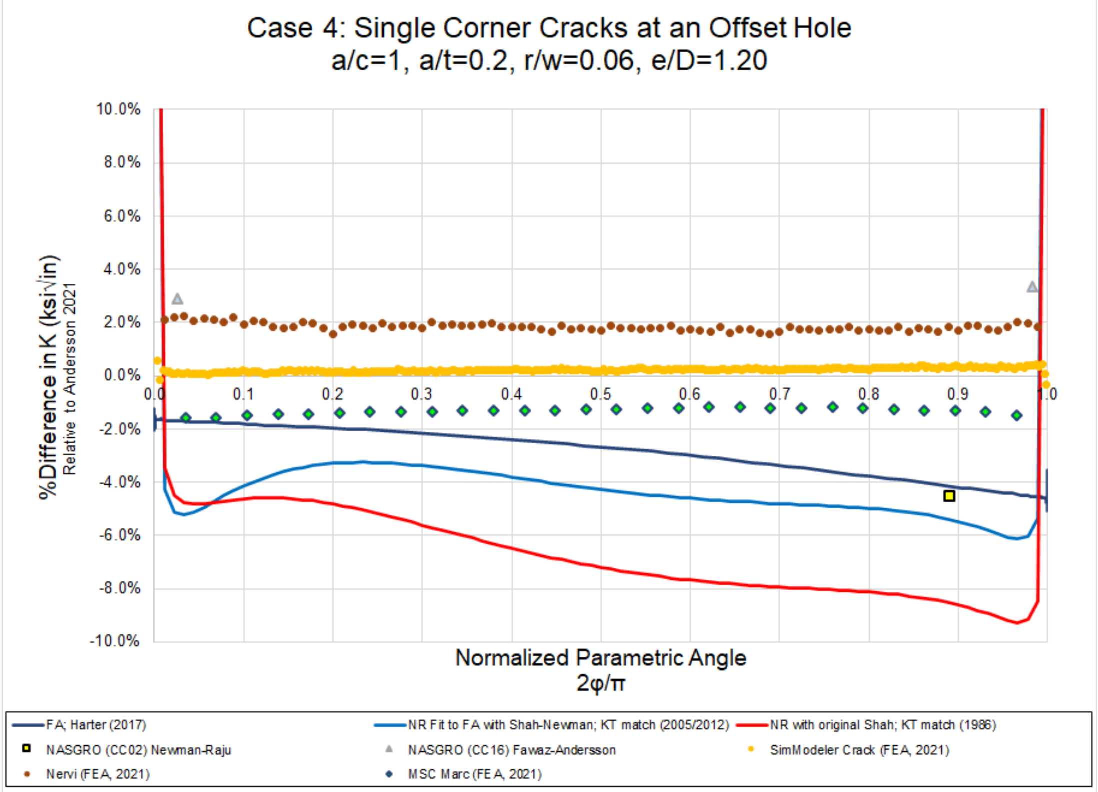


Summary: Case #4

- Case #4: Single Corner Cracks at a Hole, Finite Plate, Offset Hole
  - Continuation from Cases 1-3, incorporating hole offset effects
    - Submission 1 utilized the Harter offset correction
    - Submission 2-3 investigated two approaches to characterize the short offset, however, the Kt match approach was utilized for comparison
    - Submission 4 used the correction from [12] and Submission 5 used the correction from [13]
  - Significant differences (nearly 10% relative to Andersson submission) observed for Submissions 1-4

Case #	4
Configuration	Finite Plate, Single Crack, Offset Hole
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.050
Width	4.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	0.60
a/c	1.00
a/t	0.20
W/D	8.00
r/t	1.00
r/W	0.06

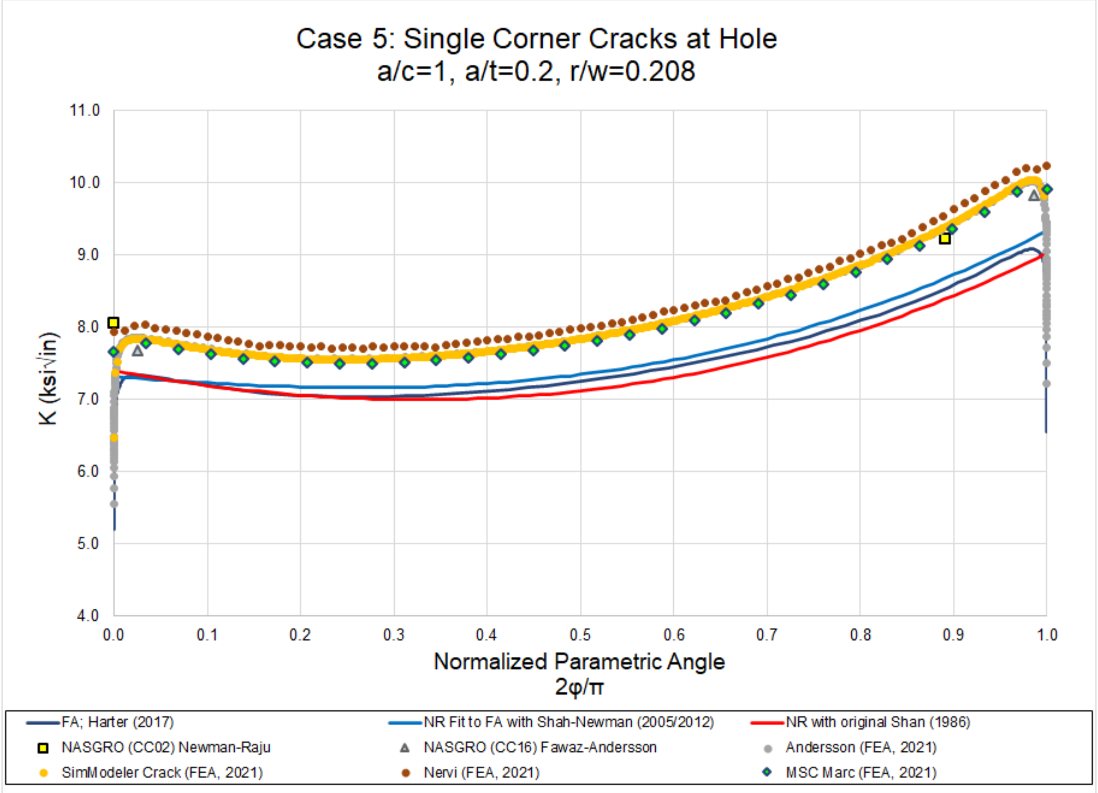


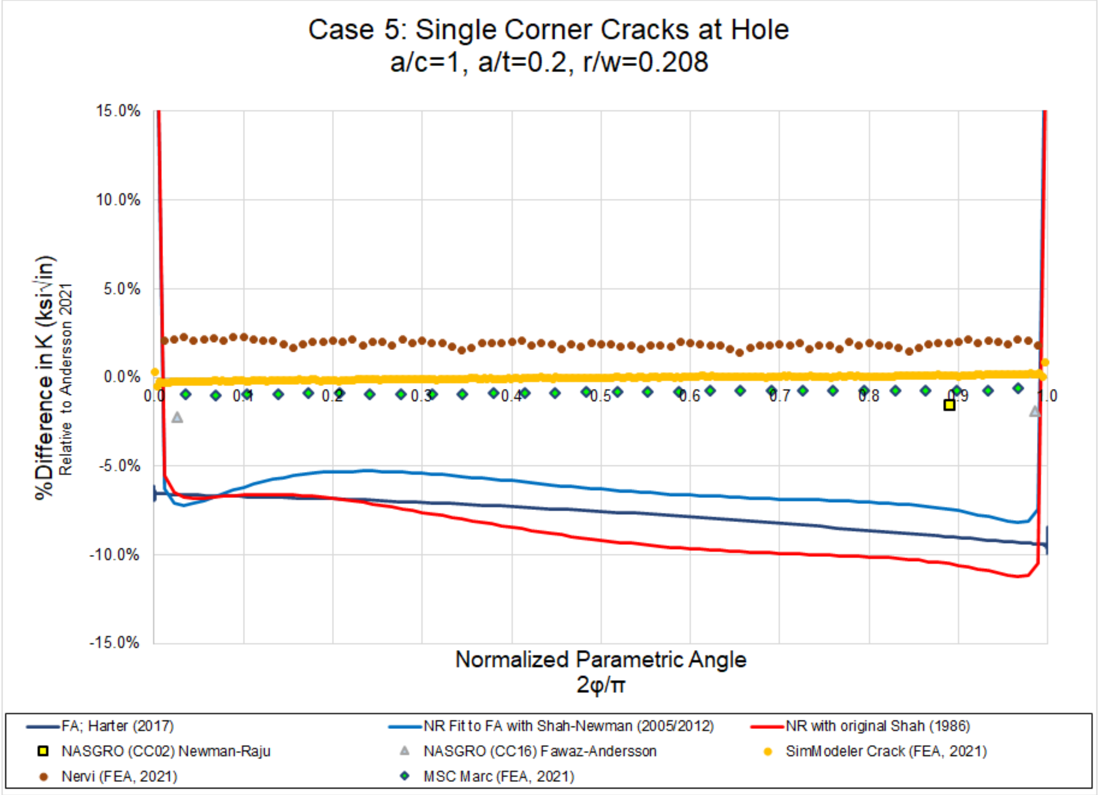


Summary: Case #5

- Case #5: Single Corner Crack at a Hole, Narrow Plate
  - Continuation from previous cases, but for relatively "narrow" width
    - Submissions 1-3 utilized the Newman finite width correction
    - Submission 4 used the correction from [12] and Submission 5 used the correction from [13]
  - Significant differences (5-12% relative to Andersson submission) observed for Submissions 1-3, which utilized Newman finite width correction

Case #	5
Configuration	Narrow Plate, Single Crack
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.050
Width	1.20
Thickness	0.25
Hole Diameter	0.50
Hole Offset	0.60
a/c	1.00
a/t	0.20
W/D	2.40
r/t	1.00
r/W	0.21

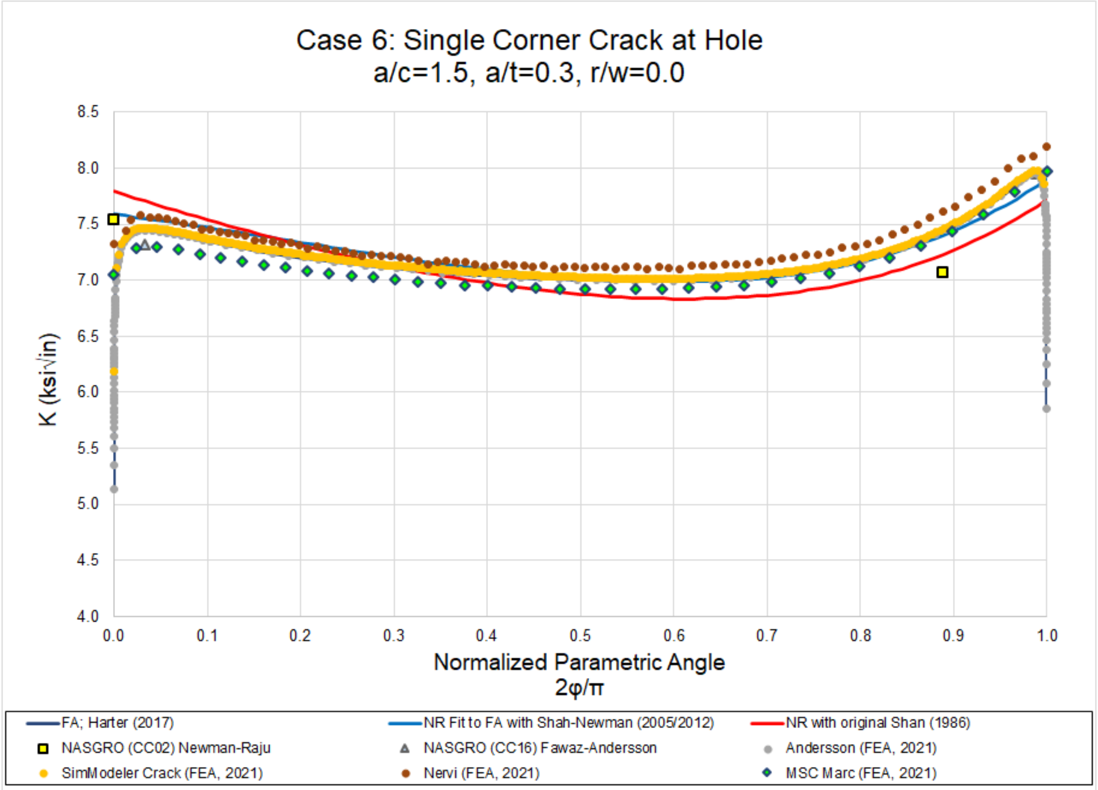


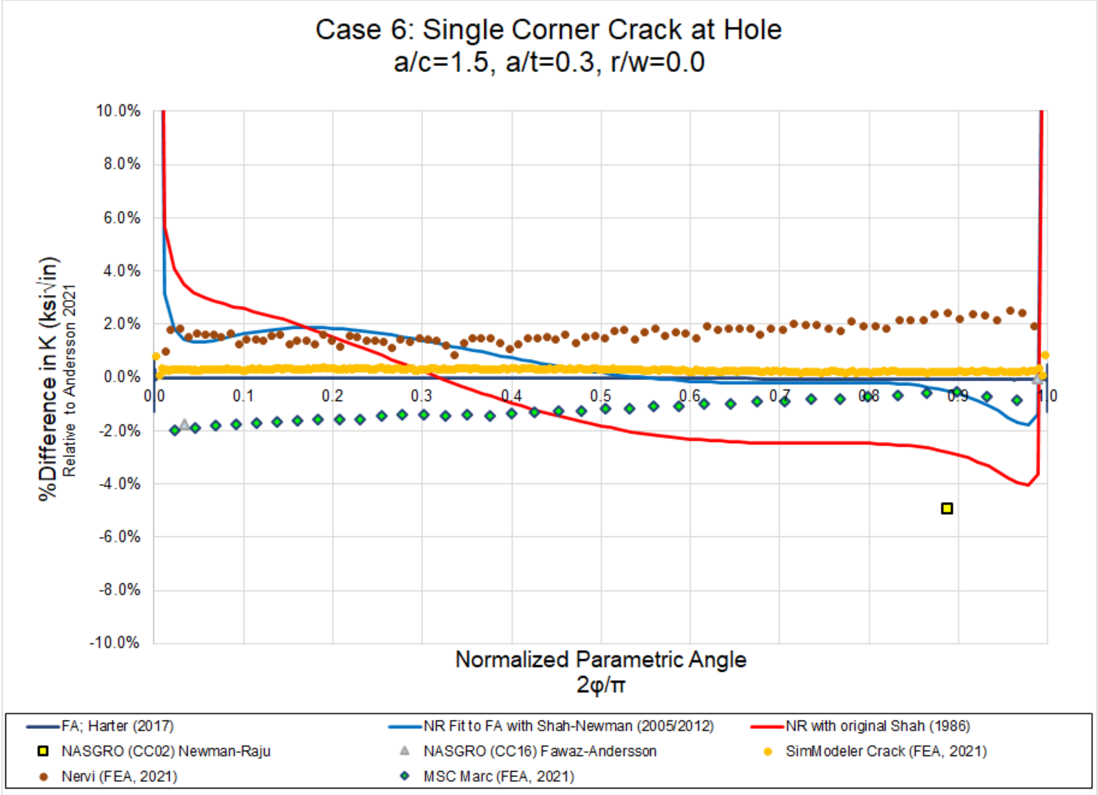


Summary: Case #6

- Case #6: Single Corner Crack at a Hole, Infinite Plate, a/c=1.5
  - Replicate of Case #2 but with a crack aspect ratio of a/c=1.5
  - Results generally within ±2% of Andersson submission, except near surface points
    - Submission 3 (Newman-Raju 1986) showed differences of ±4% across crack front
    - Submission 4 (NASGRO CC02) showed differences over 4% for point representative of hole bore

Case #	6
Configuration	Infinite Plate, Single Crack, a/c=1.5
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.050
Bore Crack Length (a)	0.075
Width	100.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	50.00
a/c	1.50
a/t	0.30
W/D	200.00
r/t	1.00
r/W	0.00





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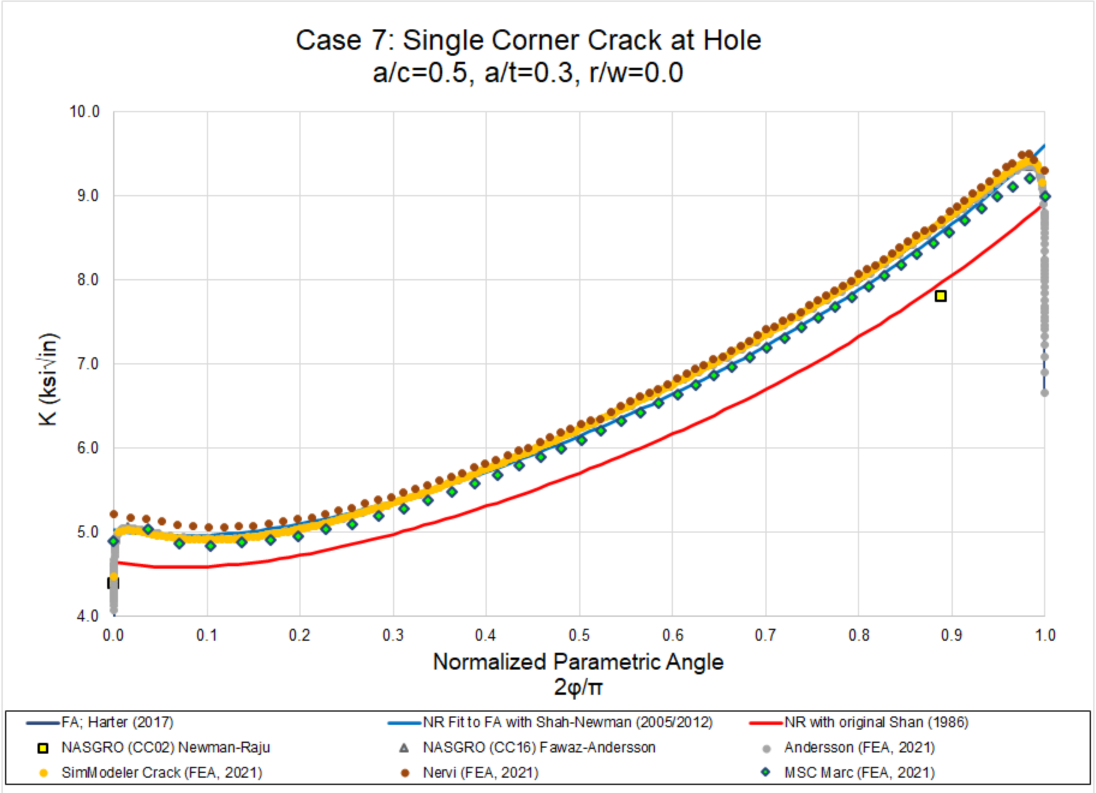
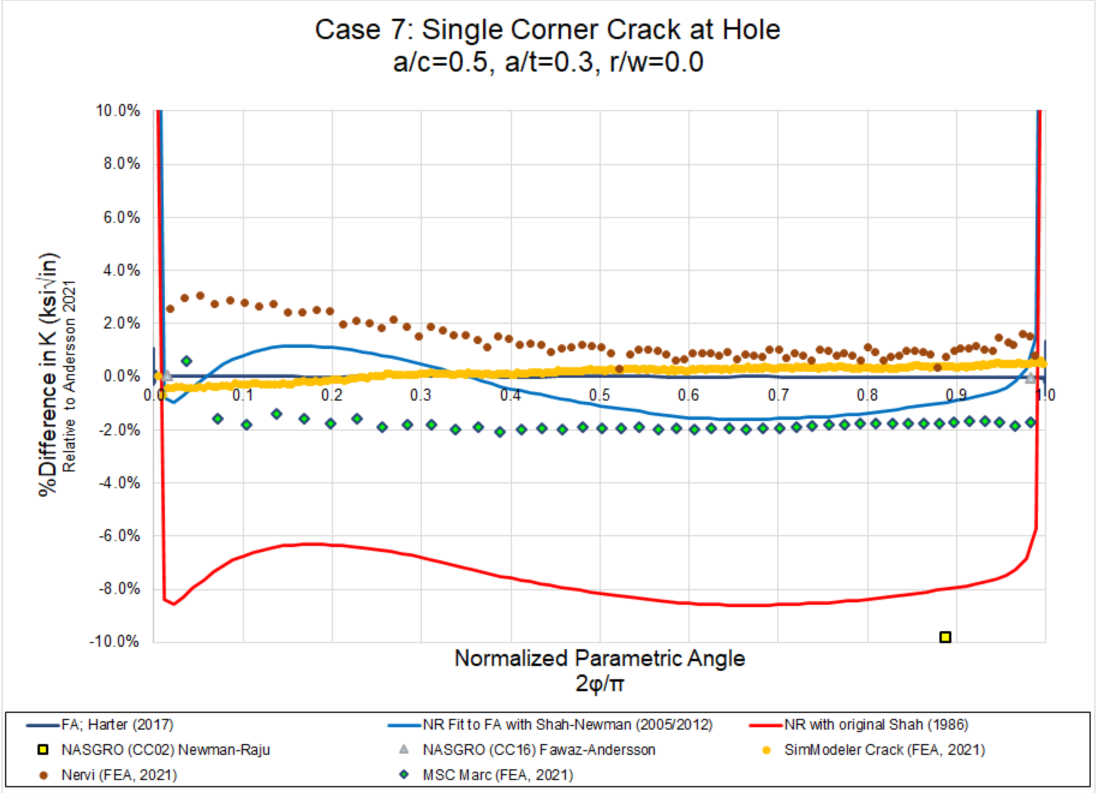
Summary: Case #7

- Case #7: Single Corner Crack at a Hole, Infinite Plate, a/c=0.5
  - Replicate of Case #2 but with a crack aspect ratio of a/c=0.5
  - Results generally within ±2% of Andersson submission, except near surface points
    - Submission 3 (Newman-Raju 1986) showed differences averaging ~8% across the crack front
    - Submission 4 (NASGRO CC02) showed differences of 10% for point representative of hole bore

Case #	7
Configuration	Infinite Plate, Single Crack, a/c=0.5
Crack Configuration	Single Corner Crack
Surface Crack Length (c)	0.100
Bore Crack Length (a)	0.050
Width	100.00
Thickness	0.25
Hole Diameter	0.50
Hole Offset	50.00
a/c	0.50
a/t	0.20
W/D	200.00
r/t	1.00
r/W	0.00





Individual Summary and Conclusions

- Submission 2: Newman-Raju Fit to Fawaz-Andersson
  - Results are within 2% for all cases except case 4 (about 6% maximum error) and case 5 (about 8% maximum error)
  - Solutions accurate to within 2% are considered acceptable for most analyses, but errors of 6-8% are larger than desired for a reliable analysis.
  - The errors for cases 4 and 5 are likely the result of limitations in the finite width correction. Improved finite width corrections are discussed in this report and are expected to improve the accuracy for the Submission 2 results.
- Submission 4: NASGRO (CC04 & CC02): Newman-Raju
  - For comparison purposes, the NASGRO (CC02) and (CC04) solutions were included in the round robin exercise, however, this solution has been superseded by CC16 and is not recommended for use anymore
  - The comparisons shown for the a-tip provide evidence to no longer use CC02 and CC04

Individual Summary and Conclusions

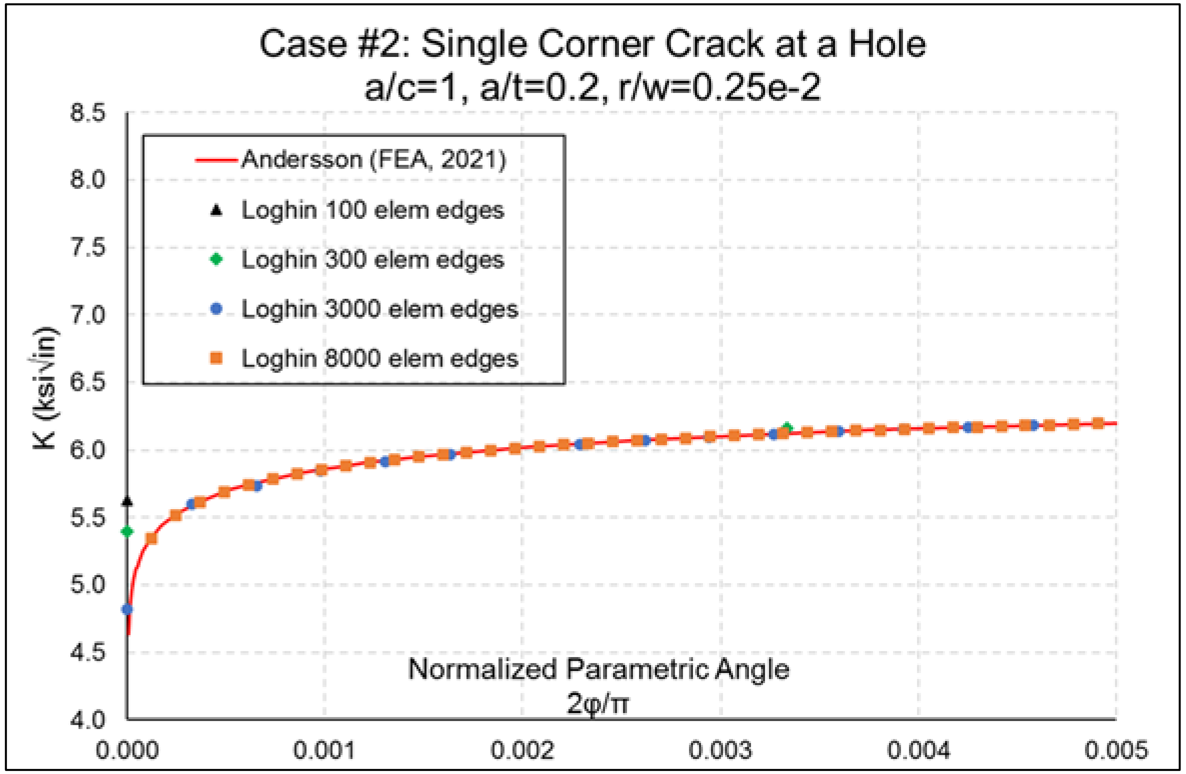
- Submission 5: NASGRO (CC16): Fawaz-Andersson
  - Parametric angles for the a-tip and c-tip CC16 SIF values were provided that correspond approximately with the maximum SIF values from the "exact" Andersson (2021) solutions
    - This is the best way to make an apple-to-apples comparison with the Andersson (2021) solutions and is consistent with the CC16 development based on the earlier Fawaz-Andersson solutions
  - These parametric angles were generally from 1 to 3 degrees from the surface depending on the case considered
  - In NASGRO, for engineering purposes, the single (local maximum) values computed at the offset angles are assigned to 0 and 90 degrees

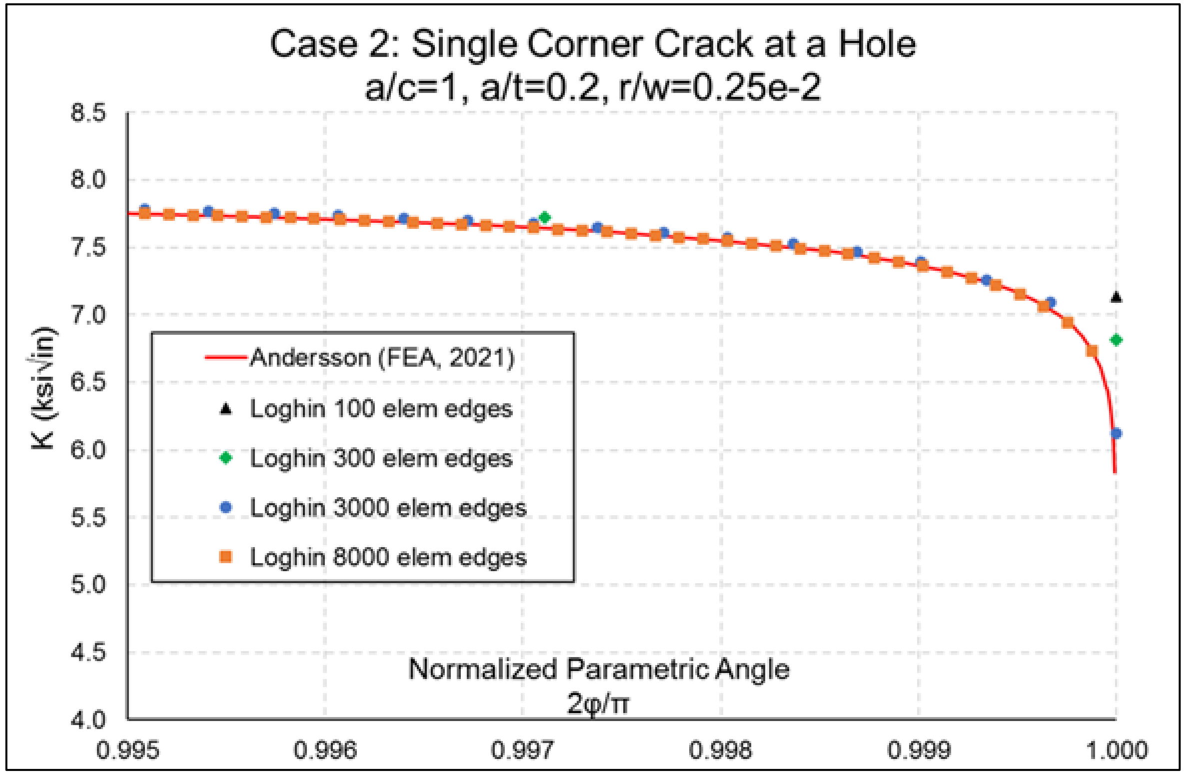
Individual Summary and Conclusions

- Submission 6: Andersson: FEA (2021)
  - BARE has delivered several 100 million accurate K-solutions to USAF Academy since 2003 which now are available in AFGROW and NASGRO, the method used for K-calculation developed 30 years ago
  - A strongly graded mesh towards and along the crack fronts are used (hp-version FEM)
    - All these meshes are designed to give relative errors of order ~0.03% along the entire crack front
  - These K-solutions are so accurate they follow the functional behavior $K=const*s^{0.04782}$
    - s being the distance to the vertex, very accurately in a large region near the vertex
    - 'const' can easily be determined in each of these millions of solutions leading to analytic formulas for K(s) near vertices
    - Note that for countersunk hole geometries K often goes to infinity near vertices
      - In such cases it is practical to express the solution near the vertex as a constant
      - For example constant=10 in $K=constant*s^{(-0.10)}$

Individual Summary and Conclusions

- Submission 6: Andersson: FEA (2021)
  - For the seven benchmark cases we used our standard procedures for K-calculation
    - The relative error in K_I in all seven solutions are then 0.03% of less
    - Made detailed convergence studies for Case #2 and compared these solutions with Loghin high accuracy solutions obtained with highly refined meshes



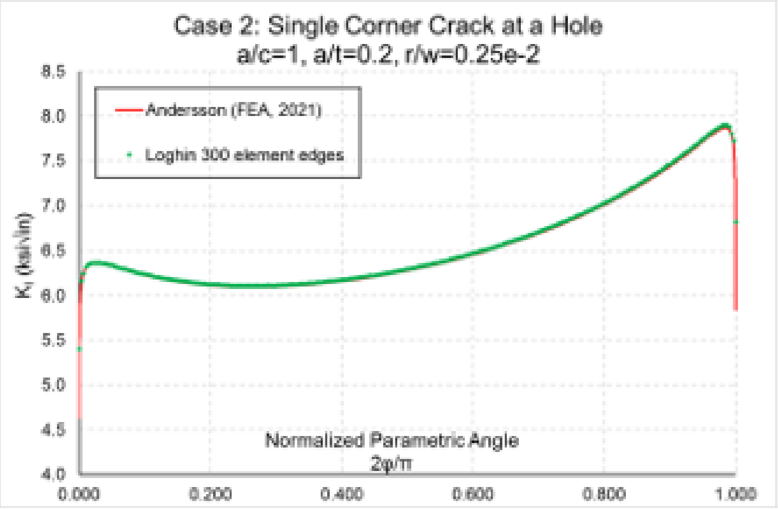


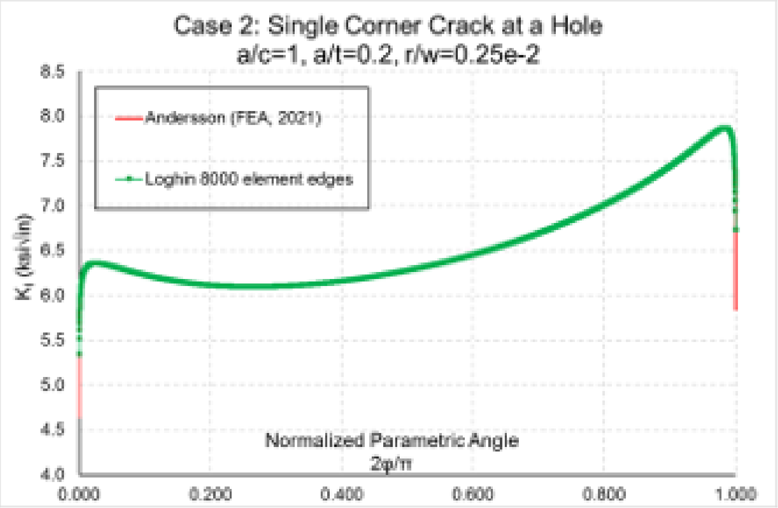
Individual Summary and Conclusions

- Submission 7: SimModeler Crack: FEA (2021)
  - SimModeler Crack was employed to create 3D finite element models and compute Mode I SIF values for the benchmark problems considered in this study
  - Any SimModeler Crack user can duplicate the results provided in this submission
  - SimModeler Crack follows an integrated CAD to FEA modeling process which allows users to easily adopt it in their design and life assessment process
  - CAD representation of each benchmark model was used to define the crack, create the associated mesh, apply boundary conditions and loading using geometric entities
  - For all cases, ANSYS was used as a solver
  - SIF values (all three modes) along each crack front are computed by SimModeler

Individual Summary and Conclusions

- Submission 7: SimModeler Crack: FEA (2021)
  - For each 3D model utilized, stress and displacement contours along with Mode I SIF for each case are provided for a detailed release of the results and a potential reference data for other studies
  - In Case 2, six models containing different mesh refinement along crack front were considered for a convergence study
    - The densest crack front mesh contains 8000 element edges
  - It was shown:
    - With increased crack front mesh density mode I SIF solution is convergent
    - The converged solution is within 0.2% average relative difference from Andersson's semi-analytical solution
    - Even though different techniques and formulations were used to compute stress intensity factors, Andersson and Loghin solutions match within 0.2%, comparison provided at right





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Individual Summary and Conclusions

- Submission 8: StressCheck: FEA (2021)
  - ESRD interpreted the objective of this round robin (evaluate differences between available SIF solutions for a single corner crack at a fastener hole with remote uniform tension loading…compared to explicit finite element modeling of each case), as establishing a reference and a framework for DaDT engineers when using these tools for computing SIFs in an industrial setting
  - Since the desired accuracy depends on the goal of the analysis, having the feedback information in the form of a reliable estimate of the relative error in the quantity of interest (QoI) is an essential technical requirement
  - In addition, users must be provided means to reduce the relative error at the expense of a reasonable amount of additional computational effort, when necessary
  - The error of approximation was evaluated from a converging sequence of solutions obtained by p-extension on a fixed mesh
  - No prior knowledge of an exact solution or the reference results provided herein were used for error estimation, such information is not available in industrial applications

Individual Summary and Conclusions

- Submission 9: MSC Marc: FEA (2021)
  - As MSC Marc is a general purpose commercial non-linear solver, focus is on ease-of-use
    - Model setup using solid geometry, both for plate and crack location
    - Model is set up in minutes
  - The crack and refined mesh is automatically created inside the Marc solver.
    - A special focused mesh with very regular mesh is used around the crack
  - VCCT was used for the SIF evaluation
    - Alternatively, the more accurate J-integral can be used
    - The mesh used is perfect for J-integral evaluation
    - No assumption is made about an exact solution
    - Standard lower order tetrahedral elements used
    - Runs in a minute on a laptop
  - Assumed that the recommended resolution of 30 elements along the crack should be used
    - Yes, it said minimum
  - Despite the relatively coarse mesh, results are within 2%
    - Thanks to the regular mesh
    - Definitely need finer mesh to resolve the solution near the ends
    - One can easily bias the edge lengths towards the ends
  - If needed, the crack can also grow, and the refined region will follow the crack

Overall Summary and Conclusions

- Successful SIF comparisons completed utilizing a wide array of available solutions and toolsets, with submissions provided by (8) different participants
- Overall, results were within 2% of the reference case, however, deviations were observed for narrow width and varying aspect ratio cases exceeded 10% in some cases
- Issues with commonly utilized finite width corrections were discovered
- A robust dataset was developed that can be utilized as a reference set for follow-on studies
- Comparisons between varying FEM approaches have highlighted the opportunity to identify modeling best practices and provide guidance to the community

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Next Steps

- Finalizing summary report documenting round robin approach, results, conclusions, and follow-on investigations
- New finite width corrections in work to support the community
- Collaboration to identify FEA best practices and lessons learned
- Consider publication of papers/presentations to share results with community

Thank you!!

Questions?

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ENGINEERED RESIDUAL STRESS IMPLEMENTATION

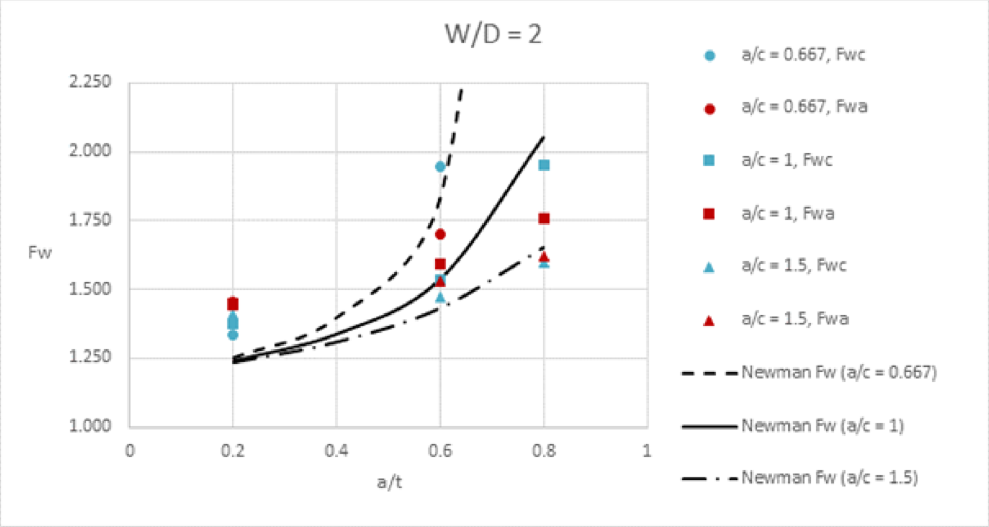
Backup Slides

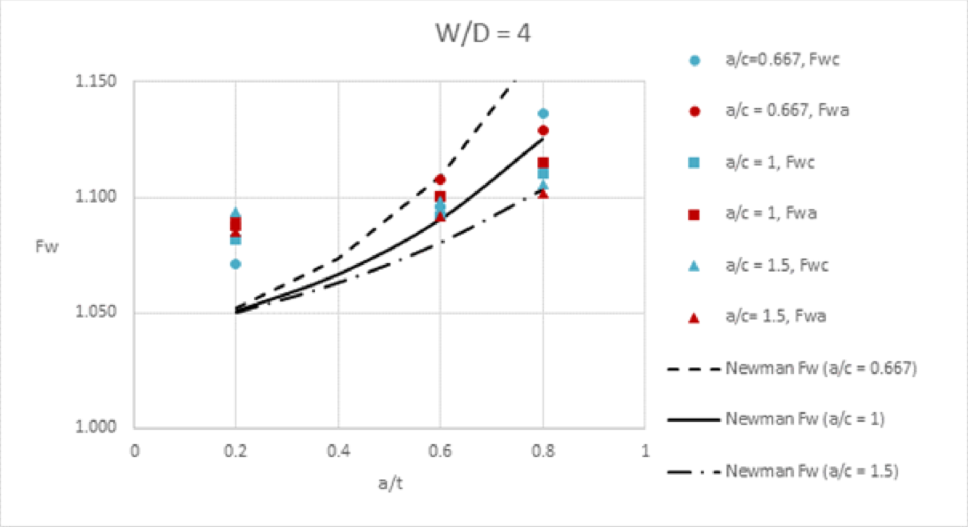
Follow-on Investigations

- Case #2 Convergence Study: Two convergence studies were carried out in parallel
  - Andersson (Submission 6)
    - Made use of an hp-formulation and showed that the original solution (Andersson FEA 2021) had a relative in K of order 0.03% or less at arbitrary points along the crack front
    - Then derived a solution with relative error ~0.01% along the entire crack front including the vertex regions
    - This was possible by utilizing the known mathematical behavior of the solution near a vertex
    - By using a least-square approximation to the calculated p=8 solution (obtained with a fine mesh) got two analytic expressions for K near and at the two vertices which are ~0.01% in error
    - By using these two analytic expressions and the p=8 solution away from vertices, have a very accurate semi-analytic expression for K, which was used for comparison with Loghin high-accuracy solutions
  - Loghin (Submission 7)
    - Made use of a built-in element formulation in ANSYS and (7) uniform meshes (100, 200, 300, 1000, 2000, 3070, 8200 element edges) along the crack front via SimModeler
    - Stress intensity factor values are computed in SimModeler using displacement correlation technique
    - The semi-analytical solution derived by Andersson was used as a reference in the convergence study to calculate relative difference for each solution
  - It is shown that, with increased mesh refinement, SimModeler solution is within 0.2% relative difference from Andersson' semi-analytical formulation

Follow-on Investigations

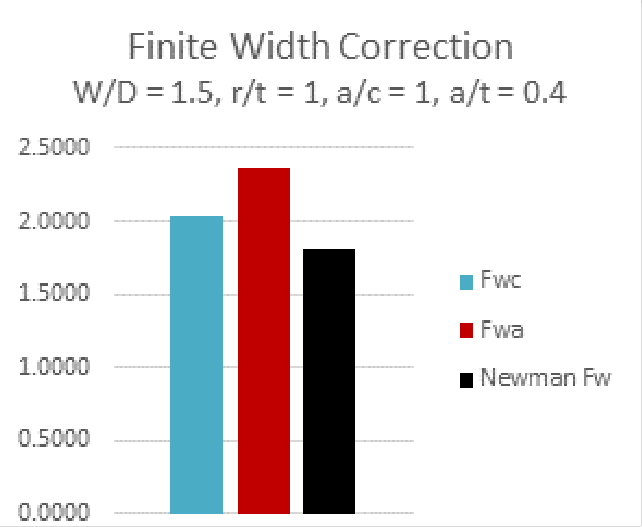
- Finite Width Correction
  - The Newman finite width correction [5] appears incorrect for Case #5
    - A comparison with FEA results (SimModeler) for a few sample cases indicate that this finite width correction is too low for cracks when a/t ~< 0.6
    - When a/t > 0.6 and W/D < 4, the correction becomes too large
    - In addition, this correction is assumed to apply equally to both crack growth directions
    - FEA results for W/D ≤ 4 (r/t = 1) indicate differences in width correction for each growth direction > 5%
    - For W/D ≤ 2, the difference becomes exceptionally large (10% – 15+%)
    - The required finite width corrections are shown with the Newman correction for these cases

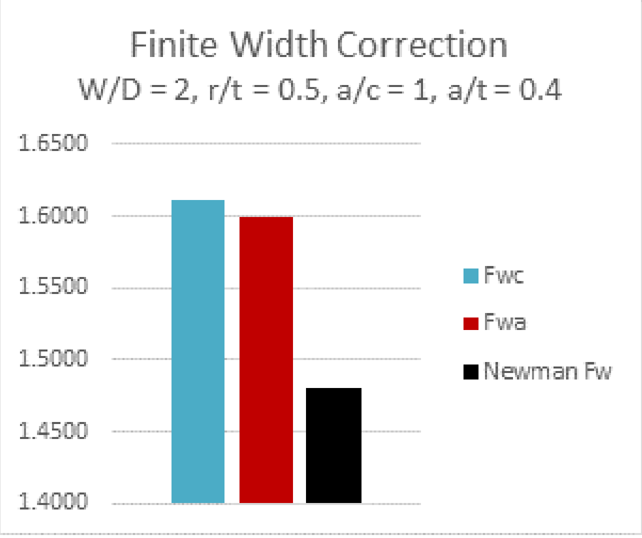




Follow-on Investigations

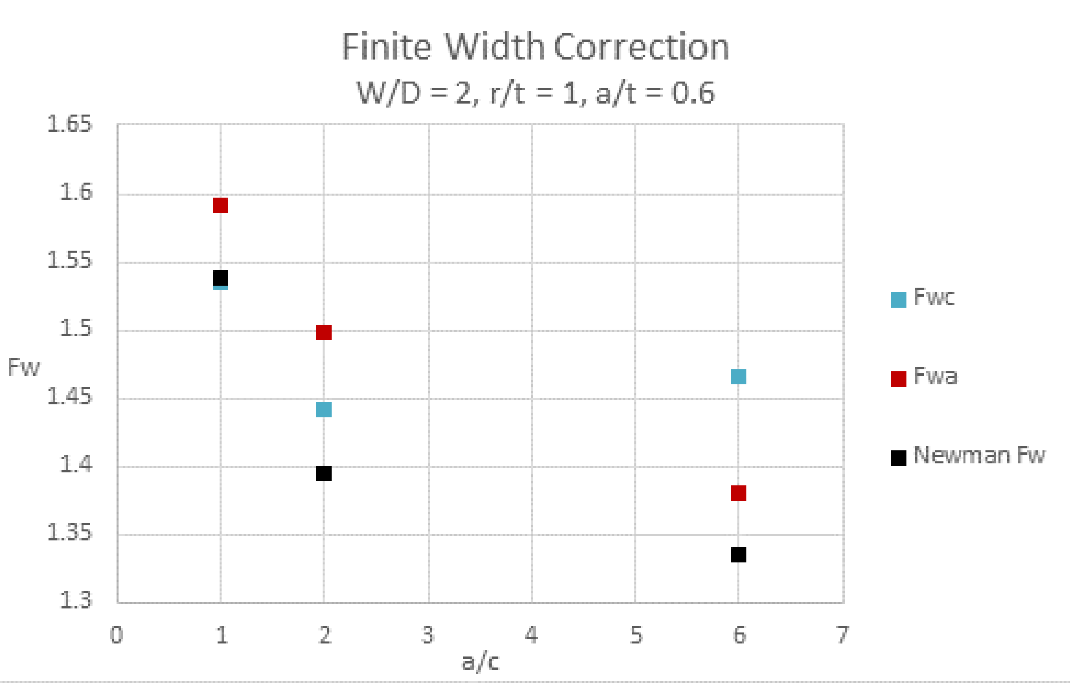
- Finite Width Correction (cont'd)
  - The Newman finite width correction [5] appears incorrect for Case #5
    - A few additional cases were analyzed using SimModeler for a different r/t and higher aspect ratio cracks to assess sensitivity of the finite width correction to other geometric parameters
    - Appears the width correction for narrow plates is a function of several parameters (i.e., a/c, a/t, and r/t)
    - The additional cases included an extremely narrow plate (W/D = 1.5), an example for a narrow plate with r/t = 0.5, and two higher aspect ratio cracks (a/c = 4 and a/c = 6)
    - Narrow plate with W/D =1.5, a/c = 1, a/t = 0.4, and r/t = 1
      - Difference in width corrections is ~17%
      - The Newman width correction is 10 – 25% too low
    - Narrow plate with W/D = 2.0, a/c = 1, a/t = 0.4, and r/t = 0.5
      - Difference in width corrections is < 1%
      - The Newman width correction is approximately 7.5% too low
    - Results indicate hole radius to thickness ratio (r/t) has significant effect on required finite width correction





Follow-on Investigations

- Finite Width Correction (cont'd)
  - The Newman finite width correction [5] appears incorrect for Case #5
    - The effect of crack aspect ratio is also very interesting
      - Differences between the finite width correction in each growth direction is relatively minor (~3-5%)
      - The Newman correction tends to be low in all cases, but the magnitude of the width correction switches from being higher in the a-direction for a/c = 1 and 2, to becoming lower in the a-direction for a/c = 6
    - This issue should be investigated thoroughly since major modifications are required to correct the closed-form finite width correction
    - In 2013, Guo [13] developed revised finite width corrections for the a-tip and c-tip values
    - Andersson has completed analyses to support an updated finite width correction
    - Solutions being utilized by Harter and Newman to generate new finite width corrections



Follow-on Investigations

- Submission 8 (StressCheck FEA) Updated Meshing Strategy & Associated Results
  - All use cases were cast on a single parametric file with guided mesh (creation of a 'parent' coarse mesh followed by automatic h-discretization)
  - The advantage of this approach is that high accuracy (for this study we targeted < 0.5%) can be achieved with minimal computational effort (< 1 min to produce a sequence of solutions of increasing number of degrees of freedom by p-extension and to extract the SIFs along the crack front for each case)
  - The computation of the SIFs performed using the Contour Integral Method described in [15]
  - Convergence in the estimated error in energy norm (global) and local convergence on SIF was obtained from the solutions computed using uniform p-extension from p-level 6 to 8
  - For more details on recommended approach for computing SIF in StressCheck refer to [14]

Follow-on Investigations

- Submission 8 (StressCheck FEA) Updated Meshing Strategy & Associated Results
  - Case 1: Double Symmetric Corner Cracks (Infinite Plate)

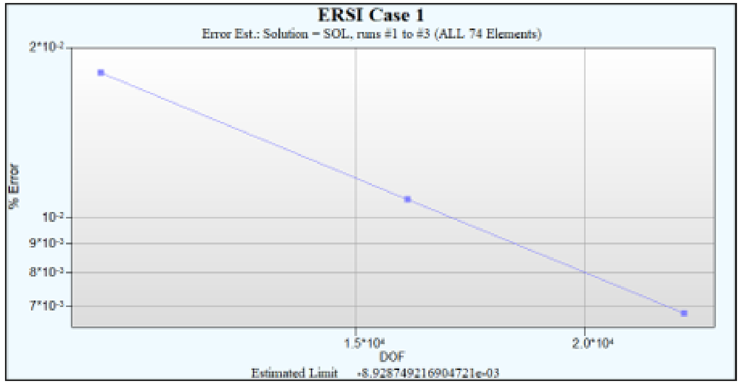


Figure 18 – Convergence of the estimated relative error in energy norm.

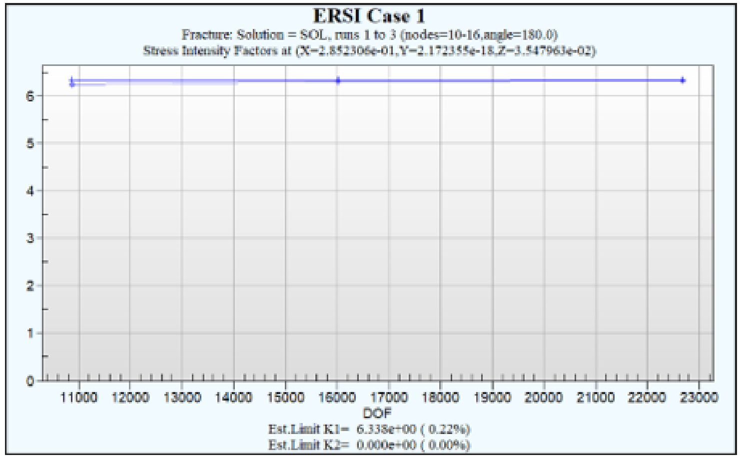


Figure 19 – Convergence of KI for an arbitrary location, for a sequence of hierarchic solutions computed with uniform p-extension (p = 6 to 8).

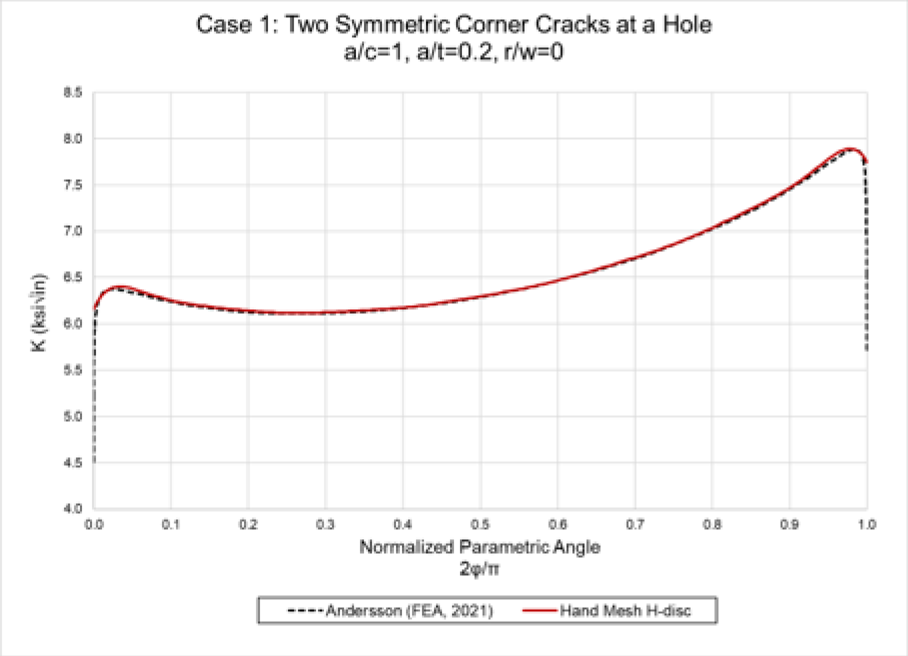


Figure 20 – Comparison of SIF extraction along the crack front with Andersson's reference solution for case 1.

Submissions

- Submission 1: Fawaz-Andersson Solutions, AFGROW
  - Utilized SIF solutions from Fawaz-Andersson [3] for a single corner crack at a fastener hole which are currently utilized in the Advanced Model solutions in AFGROW
  - To correct for a finite width, the Newman correction [5] is utilized (Cases 3-5)
  - To account for an offset hole, the Harter correction [6] is utilized (Cases 4-5)

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 2: Newman-Raju Fit to Fawaz-Andersson
  - Utilized updated equations by Newman [7] based on fit to corner crack at hole Fawaz-Andersson solutions [4]
  - To correct for a single corner crack, Shah-Newman Correction (2020) [8] was utilized (Cases 2-7)
  - To correct for a finite width, the Tada, Paris and Irwin correction [9] is utilized (Cases 3-5)
  - To account for an offset hole, two methods were investigated (Cases 4-5)
    - First option was to assume the total width was 1.2 inches with a center hole (conservative option)
    - Second option was to also assume a central hole but modify width such that K_t at the edge of the hole (short ligament side) matches the correct K_t for the offset hole geometry
      - Expected to produce more accurate solution (referred to as the K_t match approach)

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 3: Newman-Raju (1986)
  - Utilized the 1986 Newman-Raju solution [9]
  - To correct for a single corner crack, the Shah correction was utilized (Cases 2-7)
  - To correct for a finite width, the Newman correction [5] is utilized (Cases 3-5)
  - To account for an offset hole, two methods were investigated (Cases 4-5)
    - See details from Submission 2
    - The K_t match approach was utilized for comparisons

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

Submissions

- Submission 4: NASGRO (CC04 & CC02): Newman-Raju
  - Utilized the NASGRO CC04 solution, which incorporates 1986 Newman-Raju solution [9]
  - To correct for a single corner crack, the Shah correction in the NASGRO CC02 model was utilized (Cases 2-7)
  - To correct for finite width effects and an offset hole, the NASGRO CC02 model utilizes a solution for a through crack from an offset hole in a plate [12] (Cases 3-5)
  - These legacy NASGRO solutions (CC02 and CC04) were included in the round robin exercise for comparison purposes & have been superseded by CC16 and are not recommended for use anymore

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

Submissions

- Submission 5: NASGRO (CC16): Fawaz-Andersson
  - Utilized the NASGRO CC16 solution, which incorporates Fawaz-Andersson solutions [3]
    - Fundamentally based on the original Fawaz-Andersson solutions, CC16 represents the a-tip and c-tip SIFs with single values based on the local maximum (peak value) observed near the surface, which is usually around 2 to 3 degrees, but varies from case to case
  - To correct for a finite width, a modified version of the Newman finite width correction factor [5] was used [13] (Cases 3-5)
  - To account for an offset hole, the Harter correction from AFGROW [6] is utilized in NASGRO CC16 solution (Cases 4-5)
  - For the comparisons in this study, the CC16 results are compared to the Andersson (2021) local maximum results at the angles reported for the Andersson (2021) solutions

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 6: Andersson: FEA (2021)
  - Explicitly modeled each condition utilizing the STRIPE FE-software for the hp-version of the finite element method
  - Convergence tests and comparisons to the exact solution were completed for case
  - Shown that point wise relative error in K at arbitrary points along the seven crack fronts was less than 0.03%
  - This submission was utilized as the reference solution for comparisons as a result of the convergence studies

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 7: SimModeler Crack: FEA (2021)
  - Utilized SimModeler Crack to create 3D finite element models and compute Mode I SIFs via displacement correlation technique based on model solution performed in ANSYS, ABAQUS or CalculiX
    - SimModeler Crack is a pre- and post-processor designed for component level finite element-based 3D FCG simulations
  - For all models, a similar overall mesh refinement and a uniform mesh size that provides about 300 element edges along the crack front were used
    - For Case 2, an average relative difference of 0.23% from Andersson semi-analytic solution was computed, with similar differences expected for the other cases

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 8: StressCheck: FEA (2021)
  - Utilized StressCheck to create 3D finite element models and compute Mode I SIFs
    - Initial modeling approach utilized recommended modeling practices for cracked bodies, with automated graded meshing towards the crack front, targeting an estimated error in the computation of SIF within 2%
    - Additionally, the same cases were computed utilizing a completely parametric model with guided mesh targeting an estimated error in the computation of SIF ≤ 0.5%
    - The computation of the SIFs was performed using the Contour Integral Method
    - Solution verification included the estimated relative error in energy norm (global error measure) and the local convergence of SIFs at points along the crack front from a sequence of solutions obtained by uniform p-extension from p-level
    - No prior knowledge of an exact solution or the reference results provided herein were used for error estimation, such information is not available in industrial applications

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5

- Submission 9: MSC Marc: FEA (2021)
  - Utilized MSC Marc to create 3D finite element models and compute Mode I SIFs
    - Marc is a general-purpose non-linear solver with special capabilities for crack initiation and crack growth using automatic remeshing
    - The mesh for the crack evaluation was generated automatically, using the recommended minimum number of evaluation point of 30 as the number of nodes along the crack front

Case #	Configuration
1	Infinite Plate, Double Crack
2	Infinite Plate, Single Crack
3	Finite Plate, Single Crack
4	Finite Plate, Single Crack, Offset Hole
5	Narrow Plate, Single Crack
6	Infinite Plate, Single Crack, a/c=1.5
7	Infinite Plate, Single Crack, a/c=0.5