Authors: B Andersson, R. Pilarczyk and J. Guymon et al.

Round Robin Stress Intensity Factor Benchmark

Below we summarize results from a recent benchmark study "Stress Intensity K_I Comparison Round Robin" executed 2021-2022 [1]. The study was launched by the Engineered Residual Stress Implementation (ERSI) working group.

The background is that in 2017, a "Fatigue Crack Growth (FCG) Analysis Methods" round robin was completed with the objective to quantify the crack growth life for cold expanded fastener holes. During this round robin, some peculiar results found were judged to be the result of errors in K_I-solutions, however other factors could also had been contributing. As the extent of the error was unclear, further work was deemed necessary to quantify any error or discrepancy in the K_I-solutions.

As a result of these findings a follow-on collaborative round robin was established, the one summarized here, to investigate differences in stress intensity factors readily available in commercial software like AFGROW and NASGRO.

Benchmark objectives. The primary objective of the Stress Intensity Factor ($\boldsymbol{K_I}$) round robin was to evaluate differences between available $\boldsymbol{K_I}$-solutions for a single corner crack at a fastener hole with remote uniform tension loading. The evaluations included effects single versus double cracks, finite width, and hole offset. These solutions were compared to explicit Finite Element Analysis (FEM) results of each case. Any findings were intended to drive improvements to solutions available to the fracture community.

Overview of benchmark cases. The present round robin considered seven different cases of corner crack(s) at a hole in a rectangular plate, see Figure 1. Calculated $K_I(\phi)$-solutions along the crack front ($0 \leq \phi \leq \pi/2$) were requested from the eight participants in this blind test. A building block approach was utilized when setting up the seven benchmark cases in order to understand the influence of various factors. Table 1 provides an overview of the seven cases evaluated for the round robin. Case 1 represents the reference solution, without any corrections for single cracks, finite width, hole offset or crack aspect ratio. The Poisson's ratio used in all analyzes is 0.30.

Case	*Cracks*	*W/D*	*a/t*	*a/c*	*Offset/W*
1	2	200	0.1	1	0.5
2	1	200	0.1	1	0.5
3	1	8	0.1	1	0.5
4	1	8	0.1	1	**0.15**
5	1	**2.4**	0.1	1	0.5
6	1	200	0.2	1.5	0.5
7	1	200	0.2	0.5	0.5

Table 1: Summary of Round Robin Cases

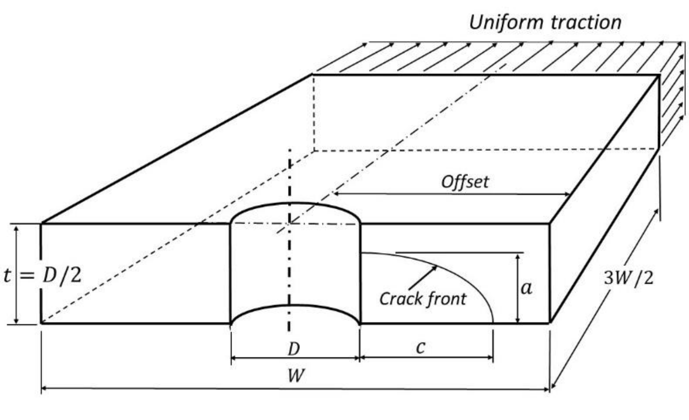

Figure 1: Round robin domain (half domain is shown) with single corner crack at a straight shank hole. The 'Offset' shown is the distance from hole center to the right specimen surface.

Round robin contributors and analysis techniques used. Table 2 provides a matrix of all submissions and analysis techniques used. There were three software companies which used the FEM-software StressCheck, Marc and SimModeler respectively to analyze Cases 1 to 7. Two software designed for fatigue crack growth predictions, namely AFGROW and NASGRO were also used. Classical Newman-Raju semi-analytic approach, including recent developments (by J. Newman [1]) were also included in the study. An USAF contractor, BARE, employed a *hp*-version of the finite element method and derived (Submission #6) highly accurate reference solutions to Cases 1 to 7 to be used as reference in the blind test. Each of these solutions had a relative error in K_I of less than 0.03% at arbitrary points ϕ along the entire crack front (including the vertex regions where $K_I \to 0$).

Benchmark results. Figure 2 exemplifies the relative error in $K_I(\phi)$ in the range $0 \leq \phi \leq \pi/2$ for Case 1 for all submissions. The figure shows that all submissions, that is FEM-solutions, AFGROW and NASGRO and the semi-analytic Newman-Raju method all gives small errors within +/- 2% except very close to the two vertices where the errors go to infinity as the reference solutions have $K_I \to 0$ at the vertices.

Table 3 which gives an overview of all benchmark results shows the obtained error ranges for Case 1 to 7 and all submissions. The table shows that the errors in the three different FEM-solutions are small for Cases 1-7 for practical purposes. An error of 2% in K_I through-out an entire fatigue crack growth analysis would typically result in an error of 10% in predicted fatigue life, except for load spectra which leads to low K_I-values close to threshold values. The errors are also small for Case 1, 2, 3, 6 and 7 in all submissions.

The only cases of concern are Cases 4 and 5 (highlighted in red color in the table) for a few submissions. The large errors for Case 4 shows that the two functions used for compensating for offset in AFGROW, NASGRO and the NR-solutions is not very accurate (the techniques used are listed in Table 2). For case 5, that is the narrow plate with *W/D*=2.4, AFGROW and the Newman-Raju solutions are very large in error.

Follow-on investigations. Three round robin partners performed additional K_I-convergence studies which are reported in detail in the final report [1] and in greatest detail in an accompanying Excel sheet [11]. Additional studies of finite width correction function were also initiated during the benchmark period. A study 2022 resulting in 86000 highly accurate plate analyses (relative error of order 0.03% along the entire crack fronts) covering a large $K_I(D/t, W/D, a/t, a/c)$-space for tension, bending and pin loading [10]. These solutions have been delivered to AFGROW- and NASGRO-developers during 2023.

Overall summary and conclusions:

- Successful SIF $K_I$comparisons were completed utilizing an array of available solutions and toolsets, with submissions provided by 8 different participants
- Overall, results were within 2% of the reference case, however, significant deviations were observed for the narrow width specimen leading to errors of up to 10%. Data in [10] which have been delivered to AFGROW and NASGRO groups covers completely the lack of data that existed at start of the benchmark
- Analysts wanting to use the closed form equation approach should strongly consider using the "Shah-Newman correction (2020)" [1] to correct for a single crack from a double symmetric crack
- A robust dataset, available as an Excel sheet [11], was developed that can be utilized as a reference set for follow-on studies
- More round robin challenges should be considered for advancing the knowledge of the entire damage tolerance design community. Simple geometries with more complex loading conditions

as well as component level geometries should be considered in future challenges.

Sub-mission #	Title	SIF solution source	Single Corner Crack	Finite Width	Offset Hole
Case	**-**	**-**	**2-7**	**3-5**	**4**
#1	Fawaz-Andersson Solutions, AFGROW	Fawaz-Andersson [2] (as implemented in AFGROW Advanced Model)	n/a	Newman [5]	Harter [3]
#2	Newman-Raju Fit to Fawaz-Andersson	Updated equations by Newman [4] based on fit to Fawaz-Andersson solutions [2]	Shah-Newman [1]	Newman [5]	Kt match approach
#3	Newman-Raju (1986)	1986 Newman-Raju solution [5]	Shah correction	Newman [5]	Kt match approach
#4	NASGRO (CC16): Fawaz-Andersson	Fawaz-Andersson solutions [2] (as implemented in NASGRO CC16)	n/a	Modified version [9] of the Newman correction [5]	Harter [3] (as impl. in NASGRO CC16)
#6	Andersson: FEA (2021)	Explicitly modeled each condition utilizing the STRIPE FE-software for the *hp*-version of the finite element method			
#7	SimModeler Crack: FEA (2021)	Utilized SimModeler Crack to create 3D FEMs and compute K_I via displacement correlation technique [9]			
#8	StressCheck: FEA (2021)	Utilized StressCheck to compute K_I [8]			
#9	Marc: FEA (2021)	Utilized Marc to create 3D FEMs and compute K_I			

Table 2: Summary of submissions and techniques used in the benchmark. Submission #5 has intentionally been left out from the present summary.

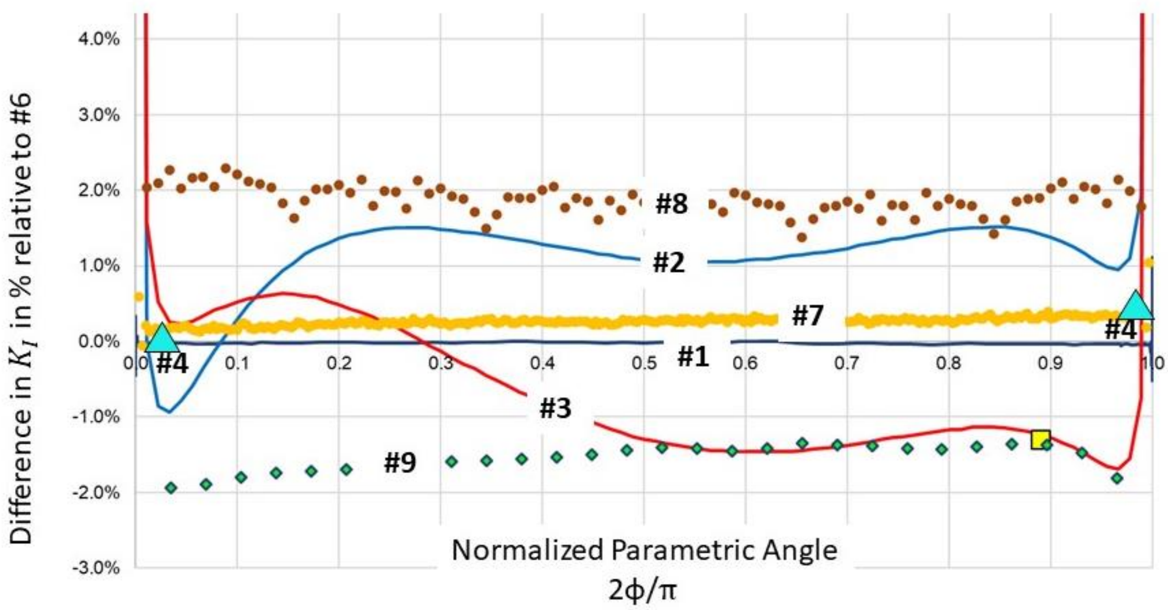

Figure 2: Relative errors in $K_I(\phi)$ for Case 1 and all submissions

Case	FEM-solutions	AFGROW	NASGRO	NR-solution
1	-2% to +2%	~0	0% to 0.5%	-1% to 2%
2	-1% to +2%	~0	-0.5% to 0%	-0.5% to 2%
3	-1% to +2%	-0.6%	-0.5% to -0.2%	-1% to 2%
4	-2% to +2%	-4.5% to -2%	+3.0% to +3.5%	-6% to -3%
5	-1% to +2%	-8% to -7%	-2%	-8% to -5%
6	-2% to +2%	~0	-2% to 0%	-2% to 2%
7	-2% to +3%	~0	~0	-2% to +1%

Table 3: Error range in K_I for Case 1 to 7 and all submissions (the close vertex regions are excluded).

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